

Nonlocal behaviour of the electron component in nonequilibrium plasmas

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Abstract: The spatial relaxation of the velocity distribution function and of relevant macroscopic quantities of the electrons is investigated in weakly ionized plasmas. In particular, the relaxation of plasma electrons in a uniform electric field, the response of the electrons on spatial disturbances of the electric field and the electron kinetics in the case of space charge field confinement are analysed. The kinetic studies are based on the numerical solution of the inhomogeneous Boltzmann equation of the electrons including the action of a nonuniform electric field and of elastic and inelastic collisions of the electrons with gas atoms. A distinctly nonlocal behaviour of the electrons and unexpectedly large relaxation lengths become evident at medium electric fields.

INTRODUCTION

In conventional discharge plasmas various discharge regions are known, in which a particular spatial structure of the plasma occurs and distinctly nonlocal properties of the electrons are of particular importance for the understanding of the plasma behaviour. In this context it should be mentioned, for example, the discharge regions in front of electrodes, the plasma regions in the surrounding of enforced plasma constrictions or of inserted grids, in moving and standing striations, and the plasma regions close to isolating walls which confine the plasma. Concerning the kinetics in electrode regions, which are generally characterized by large electric fields with rapid change in space, several investigations (1,2) have already been performed in the past. With respect to the other mentioned discharge regions some tasks (3), with more or less success, have been undertaken to reveal and to understand the electron kinetics and the global plasma behaviour in spatially structured plasmas. It is the objective of this paper to contribute by theoretical studies on the electron kinetics in nonuniform plasma regions to a better understanding of the spatial behaviour of the electron component. One important aspect of the electron kinetics in nonuniform plasma regions concerns the spatial relaxation properties of the electrons. To model such plasmas with respect to the electron kinetics often rough simplifications, as the local field approximation, have been adopted. In order to illustrate the pronounced nonlocal behaviour of the electron component detailed results on their kinetics will be reported for (i) the relaxation of plasma electrons injected into uniform electric fields, (ii) the response of plasma electrons on spatial disturbances in the electric field, and (iii) the electron kinetics in case of space charge field confinement in a dc plasma column of a glow discharge.

BASIC ASPECTS OF THE KINETIC TREATMENT

The basic equation to study the electron kinetics in nonuniform plasma regions is the inhomogeneous Boltzmann equation of the electrons

$$\vec{v} \cdot \nabla_{\vec{r}} f - \frac{e_0}{m} \vec{E} \cdot \nabla_{\vec{v}} f = C^{el}(f) + \sum_k C_k^{in}(f)$$

including the action of elastic (C^{el}) and conservative inelastic (C_k^{in}) electron atom collisions and the impact of a nonuniform electric field \vec{E} . If the field can be chosen to be parallel to a fixed space

direction, which is appropriate to the first two problems (i) and (ii), the velocity distribution becomes symmetrical around the field direction $\vec{E} = E(z)\vec{e}_z$ and can be given an expansion in Legendre polynomials. When truncating this expansion, then in the well known two term approximation

$$f(U, \frac{v_z}{v}, z) = \frac{1}{2\pi} \frac{1}{(2/m)^{3/2}} \left[f_0(U, z) + f_1(U, z) \frac{v_z}{v} \right], \quad U = \frac{m}{2} v^2, \quad v = |\vec{v}|$$

the equation system

$$\begin{aligned} \frac{\partial}{\partial z}(U f_1) - e_0 E(z) \frac{\partial}{\partial U}(U f_1) + \frac{\partial}{\partial U}(C(U) f_0) + \sum_k F_k(U) f_0 &= \sum_k F_k(U + U_k^{in}) f_0(U + U_k^{in}, z), \\ \frac{\partial}{\partial z} f_0 - e_0 E(z) \frac{\partial}{\partial U} f_0 + H(U) f_1 &= 0 \end{aligned}$$

for the isotropic and anisotropic part $f_0(U, z)$ and $f_1(U, z)$ of the velocity distribution function with the coefficients $C(U) = -6(m/M)U^2 N Q^d(U)$, $F_k(U) = 3UNQ_k^{in}(U)$ and $H(U) = NQ^d(U) + \sum_k NQ_k^{in}(U)$ can finally be derived from the inhomogeneous kinetic equation (4). Here are Q^d and Q_k^{in} total cross sections for momentum transfer in elastic collisions and for various inelastic collision processes, m and M the mass of electrons and atoms and N the density of atoms. This system of two partial differential equations with additional terms of shifted energy variable due to the energy loss U_k^{in} in inelastic collisions determines the evolution of both parts of the velocity distribution with the kinetic energy U of the electrons and the space coordinate z . A transformation from the kinetic energy to the total energy and the elimination of the anisotropic distribution leads finally to a parabolic partial differential equation with additional terms of shifted energy variable. This equation describes the evolution of the isotropic distribution and has to be solved as an initial boundary value problem on a nonrectangular solution region whose boundaries are partly determined by the special course of the electric field considered. The problem has to be completed by appropriate boundary conditions. An efficient solution approach could be developed to solve numerically this initial boundary value problem with high accuracy for various plasmas and courses of the electric field. An impression of the physics involved in the kinetic treatment can be obtained when considering the macroscopic balance equations of the electrons which are consistent to the kinetic equation. These are, in particular, the energy and the momentum balance

$$\frac{d}{dz} j_u(z) = \langle U^f \rangle(z) - \langle U^{el} \rangle - \sum_k \langle U_k^{in} \rangle, \quad \frac{2}{3m} \frac{d}{dz} \langle U \rangle(z) = \langle I^f \rangle(z) - \langle I^{el} \rangle(z) - \sum_k \langle I_k^{in} \rangle(z).$$

All macroscopic quantities occurring in these balances are obtained by appropriately energy space averaging over the isotropic and anisotropic distribution, respectively. Due to the energy balance the difference between the energy gain from the electric field $\langle U^f \rangle$ and the energy losses $\langle U^{el} \rangle$ and $\langle U_k^{in} \rangle$ in elastic and inelastic collisions is compensated for at each space position z by the spatial divergence of the energy current density j_u . The momentum balance possesses a similar structure. According to this balance the local difference between momentum input $\langle I^f \rangle$ from the field and momentum losses $\langle I^{el} \rangle$ and $\langle I_k^{in} \rangle$ by elastic and inelastic collisions is compensated for by the divergence of the mean energy density $\langle U \rangle$ of the electrons being additionally multiplied by the factor $2/(3m)$. Thus in almost homogeneous states an almost complete compensation of the input from the electric field and the loss by all collision processes should occur at each position z in both balances. Therefore such a situation is well characterized by a small divergence term compared to the gain and loss terms in each of the balances. A large simplification of the kinetic treatment of inhomogeneous plasma regions is reached when the spatial evolution of the electron kinetic quantities only consists of a sequence of almost homogeneous states. These states can be determined by solving the much simpler homogeneous kinetic equation for the sequence of the electric field strength values which are assumed by the field course considered. Such a treatment is often called as "local field approximation". In this case a very rapid spatial establishment of the electrons into homogeneous states should occur and the corresponding relaxation length of the electrons should be very short.

RELAXATION IN UNIFORM ELECTRIC FIELDS

Main aspects of the spatial relaxation of electrons in nonisothermal collision dominated plasmas can already be revealed when studying the spatial evolution of electrons which are stationarily injected into a plasma at a certain position and which are acted upon by a uniform electric field (5). Sufficiently far from the injection region finally a homogeneous state should be established in field acceleration direction of the electrons. For such a study electrons are injected at the position $z = 0$ in acceleration direction $z > 0$ with a Gaussian form of their anisotropic distribution which lastly describes the energy distribution of the electron particle current density. Figure 1 illustrates for a helium plasma at the two normalized field strengths $E/p = -4$ and -10 V/(torr cm) (p – gas pressure) the spatial evolution of the isotropic distribution normalized on the electron density $n(\infty)$ in the established homogeneous state. With increasing normalized spatial coordinate zp a pronounced and very different

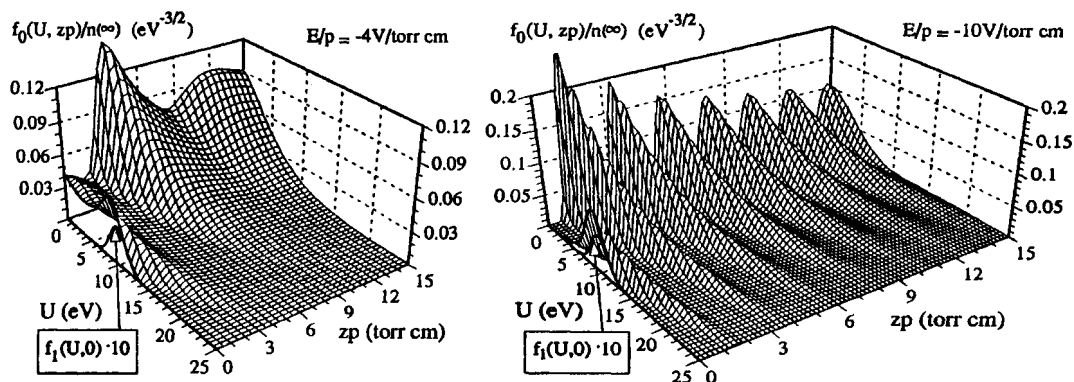


Fig. 1: Isotropic distribution as a function of the kinetic energy U and the spatial coordinate zp .

evolution of the isotropic distribution can be observed at the two field strengths. At the larger field strength magnitude a distinctly periodical structure of the isotropic distribution occurs and even after a normalized relaxation distance of 15 torr cm no establishment into the homogeneous state is reached. In particular a systematic shift of the distribution towards higher kinetic energies and a periodic occurrence of low energy electrons can be seen with increasing space coordinate. Figure 2 shows for the larger field strength magnitude the corresponding spatial evolution of all contributions to the energy balance of the electrons normalized on the space independent energy gain from the electric field. This figure clearly indicates that even after a distance of 30 torr cm still a remarkable difference

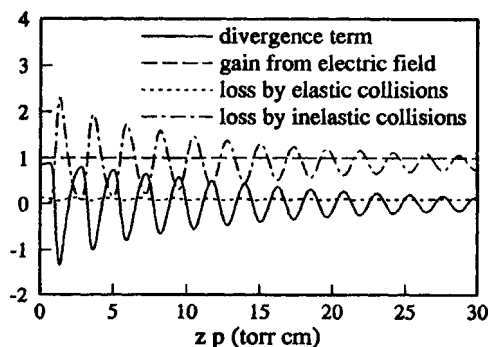


Fig. 2: Normalized energy balance terms for $E/p = -10$ V/(torr cm).

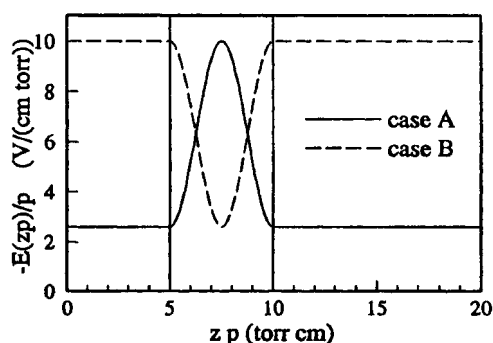


Fig. 3: Spatial course of the electric field for case A and B.

between the energy gain from the electric field and the energy loss in all collision processes remains and the divergence of the energy current density of the electrons still significantly contributes to the inhomogeneous energy balance. Thus already these few results lead to the conclusion that, depending on the special plasma conditions considered, quite different and unexpectedly large relaxation lengths of various electron kinetic quantities occur in collision dominated plasmas even under the action of uniform electric fields. If the local field approximation would be applicable to these spatial relaxation processes a very rapid spatial establishment into the corresponding homogeneous state should occur immediately behind the injection position $z = 0$.

RESPONSE ON SPATIAL FIELD DISTURBANCES

A spatially limited disturbance of the axial electric field strength can be caused, for example, in an axially homogeneous column of a dc glow discharge by special discharge arrangements. Sufficiently far from this field disturbance region axially homogeneous states towards both electrode regions should be established in the plasma column. The response of the electrons on such field disturbances has been studied in nonisothermal plasmas (4). By using a similar kinetic treatment as outlined above this spatial relaxation problem can be investigated. Contrary to the relaxation problem of injected electrons acted upon by a uniform electric field now sufficiently far away from the field disturbance region the homogeneous electron velocity distribution in the undisturbed electric field represents one boundary condition of the kinetic problem and the electric field becomes space dependent in the field disturbance region. Figure. 3 illustrates two field disturbances (case A and B), which have been

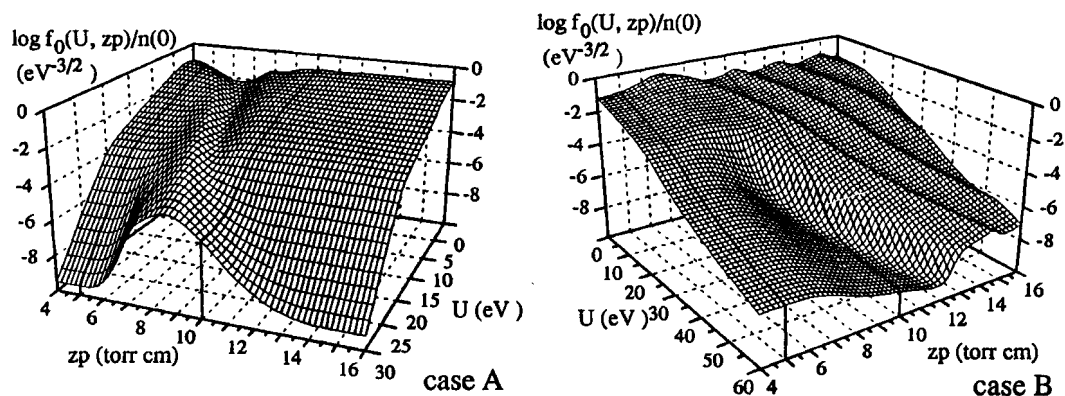


Fig. 4: Isotropic distribution as a function of the kinetic energy and the spatial coordinate.

assumed to occur both in the space region between 5 and 10 torr cm. In case A a larger and in case B a smaller magnitude of the electric field acts on the electrons in the field disturbance region compared to the undisturbed field regions. The electron acceleration by the electric field always occurs in the positive z direction. Figure 4 shows for both cases the spatial evolution of the isotropic distribution in a helium plasma starting from homogeneous states in the velocity distribution at the position $z = 0$. The field disturbance region is marked by black lines on the distribution surface. In both cases a strong response of the isotropic distribution on the field impulse can be observed with a particularly large extension of this response in electron acceleration direction. The isotropic distribution develops in case A in an aperiodic and in case B in a periodic way towards its homogeneous state at large zp . In order to illustrate the strong violation of the local field approximation under the action of such a field disturbance Fig. 5 compares at several selected kinetic energies the spatial evolution of the isotropic distributions as obtained by the strict solution of the inhomogeneous kinetic equation and by the local field approximation. Discrepancies up to some orders of magnitude are found especially in the field disturbance region (marked by vertical lines) and in a large region towards higher zp values. These results underline once more that large relaxation lengths of the electrons are responsible for the large spatial delay of the isotropic distribution response on the impulse-like field disturbance.

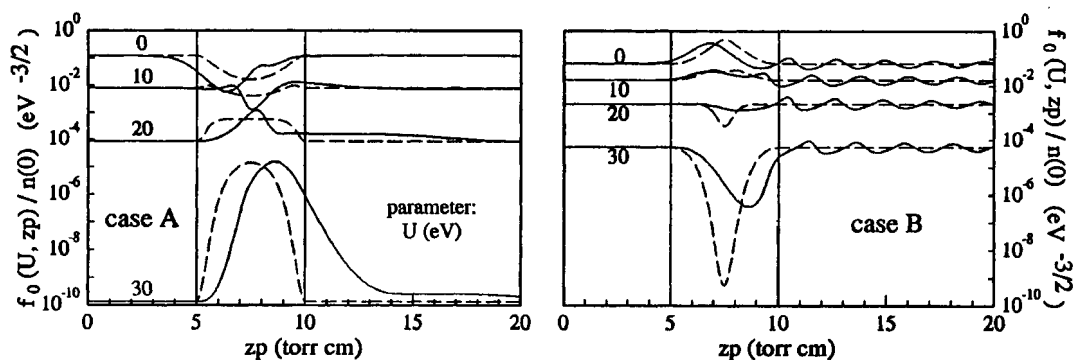


Fig. 5: Isotropic distribution functions as obtained by the solution of the inhomogeneous Boltzmann equation (—) in comparison with those from the local field approximation (- - -).

ELECTRON KINETICS IN CASE OF SPACE CHARGE FIELD CONFINEMENT

A very different situation with respect to the spatial behaviour of the electron component arises if the inhomogeneity of the plasma is caused by the plasma confinement. A typical example for this problem is given by the radial structure of the positive column in a dc glow discharge with isolating walls. This problem has already been attacked with more or less success several times in the past. From the electron kinetic point of view this problem is more complex (6) than the first two kinetic problems considered above. The electric field in the column plasma is a superposition of the constant axial electric field and of the radially varying radial space charge field, i.e. $\vec{E} = E_r(r)\vec{e}_r + E_z\vec{e}_z$. Thus the direction of the total electric field is different from the radial direction in which the inhomogeneity of the plasma column occurs. Then the expansion of the velocity distribution in Legendre polynomials has to be replaced by an expansion in spherical harmonics. The latter reads in two term approximation

$$f(U, \frac{\vec{v}}{v}, r) = \frac{1}{2\pi} \frac{1}{(2/m)^{3/2}} \left[f_0(U, r) + f_r(U, r) \frac{v_r}{v} + f_z(U, r) \frac{v_z}{v} \right], \quad U = \frac{m}{2} v^2, \quad v = |\vec{v}|$$

and includes, in addition to the axial component $f_z(U, r)$, a radial component $f_r(U, r)$ of the anisotropic distribution. This radial anisotropic distribution makes possible the calculation of the radial particle and energy current density of the electrons and allows thus to reveal significant aspects of the electron confinement by the radial space charge field. In order to describe the electron loss to the tube wall, the electron production in the column plasma by ionization in electron collisions with ground state and excited atoms has additionally to be considered in the kinetic equation. When introducing the above expansion into the kinetic equation and when replacing the kinetic energy U by the total energy finally an elliptic partial differential equation with additional terms of shifted energy variable due to the inelastic collision processes is obtained for the isotropic distribution which has to be completed by appropriate boundary conditions. The accurate numerical solution of this elliptic problem for specific plasma conditions has been found to be a laborious task. Using measured values of the axial field strength and of the radial potential distribution for the neon dc plasma column as input parameters of the kinetic problem, the radial variation of the velocity distribution and of the various macroscopic quantities of the electrons in the column plasma has been calculated by solving the elliptic problem (6). To give an example, the cylindrical column plasma of a neon discharge with a tube radius of 1.7 cm, a gas pressure of 100 Pa and a discharge current of 10 mA has been chosen. Figure 6 shows the calculated radial variation of the isotropic distribution which has been normalized on the electron density $n(r)$ at each radial position. A pronounced radial variation of the normalized isotropic distribution can

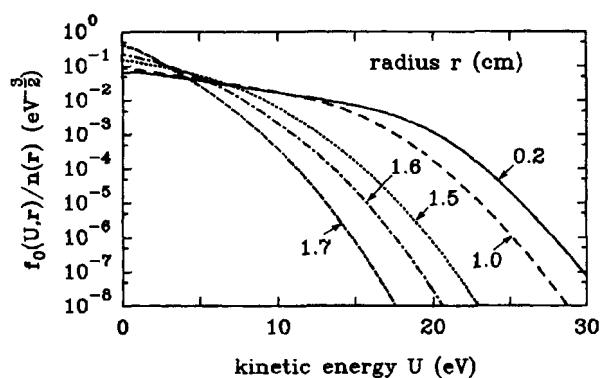


Fig. 6: The normalized isotropic distribution $f_0(U, r)/n(r)$.

be observed from this figure which is mainly caused by the interplay of the action of the radial space charge field and the collisional dissipation of the electrons in the column plasma. According to the conventional ("local") approach the normalized isotropic distribution is approximately determined by the homogeneous kinetic equation with the sole inclusion of the axial field impact and is treated thus as independent of the radial coordinate. Contrary to this approximation the consequent kinetic treatment leads to a pronounced radial variation of the normalized isotropic distribution due to the confinement by the radial space charge field. On the basis of the strict kinetic treatment of the electron confinement in addition to the isotropic distribution the determination of the axial and radial components of the anisotropic distribution became possible. Figure 7 compares for two radial positions the energy dependence of all three distribution components. In particular the relatively small

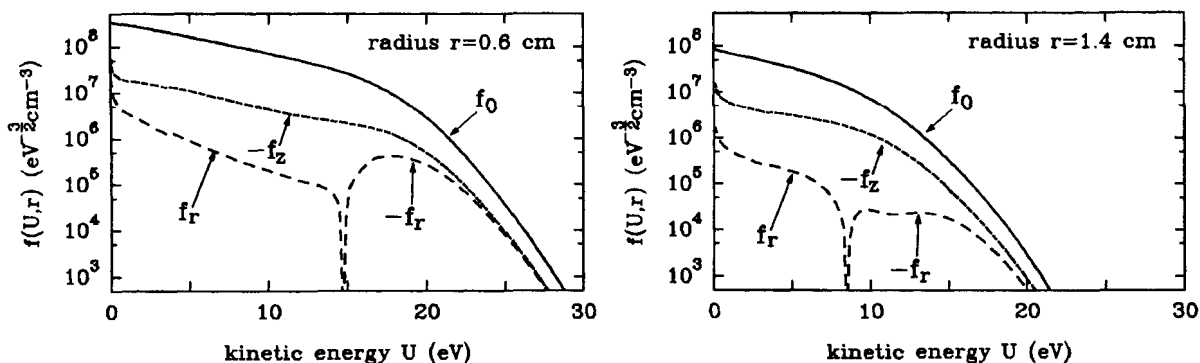


Fig. 7: The isotropic and the radial and axial anisotropic distribution $f_r(U, r)$, $f_z(U, r)$, and $f_0(U, r)$.

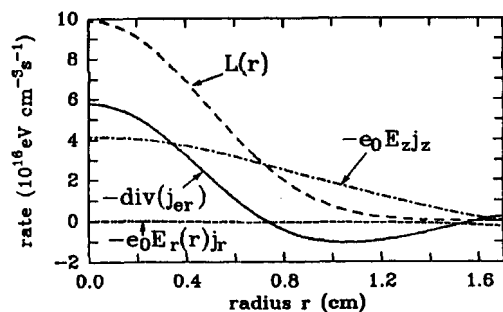


Fig. 8: The various contributions to the energy balance of the electrons.

magnitude of the radial anisotropic distribution and its two branches with different sign reflects the remarkable space charge field confinement of the electrons in radial direction. The change of the sign in the radial anisotropic distribution with increasing kinetic energy enforces a strong compensation in the radial particle current density j_r of the electrons when evaluating this quantity from the radial anisotropy. The distinctly nonlocal behaviour of the electron component in the column plasma can be illustrated from the energetic point of view too. Figure 8 shows the radial dependence of all contributions to the energy balance of the electrons. The short-long dashed line gives the energy gain $-e_0 E_z j_z$ by the axial field and the long dashed line the total energy loss $L(r)$ by all collision processes. In the central region of the column plasma the collisional energy loss is about two times the energy gain from the axial field. However outside the central region increasingly the reverse situation occurs. The large difference between the energy gain from the axial field and the energy loss by collisions at each radial position is compensated for by a large divergence of the radial energy current density j_{er} of the electrons which is presented by the full line. The very small contribution $-e_0 E_r(r) j_r$ of the radial space charge field to the energy balance is given by the short dashed line and represents a small cooling of the electrons in the space charge field.

CONCLUSIONS

A strict kinetic treatment of the electrons in spatially nonuniform regions of weakly ionized, collision dominated plasmas at medium electric field strength makes evident that a distinctly nonlocal behaviour of the electron component occurs in various plasma conditions. This nonlocal behaviour is reflected, for example, by large spatial relaxation lengths for the establishment into homogeneous states, by a pronounced spatial delay of the electron response on spatially varying electric fields and by a large transport of electron energy from the outer to the inner plasma region in the case of space charge field confinement. Rough approximations as the "local field approximation" are quite unable to describe the real situation and should be critically considered when used in plasma modelling.

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