# CONVERSION OF TEMPERATURES AND THERMODYNAMIC PROPERTIES TO THE BASIS OF THE INTERNATIONAL TEMPERATURE SCALE OF 1990 

(Technical Report)

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# Conversion of temperatures and thermodynamic properties to the basis of the International Temperature Scale of 1990 (Technical Report) 


#### Abstract

Tables of temperature differences between the International Temperature Scale of 1990 (ITS-90) and earlier temperature scales (IPTS-68, EPT-76, IPTS-48, and ITS-27) are presented. These tables also contain values of the derivatives of these differences with respect to temperature. Analytical equations to reproduce the temperature difference ( $T_{90}-T_{68}$ ) and its first derivative are also given. This information is needed for the adjustment of thermodynamic results to the basis of the ITS-90. Thus, for the most accurate thermodynamic results, it is preferable to change the temperatures of the original work to the ITS-90 and then recalculate the thermodynamic results on this basis. However, conversion formulae based upon a Taylor expansion of the enthalpy have been derived previously by Douglas (J. Res. Natl. Bur. Stand., Sect. A 73, 451-470 (1969)). These equations are greatly simplified when the differences between the two temperature scales are small. Approximate effects resulting from the conversion from the IPTS-68 to the ITS-90 and from the IPTS-48 to the ITS-90 for existing calorimetric determinations of heat capacity, enthalpy, and entropy have been calculated with the equations of Douglas for $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s}), \mathrm{BaSnF}_{4}(\mathrm{~s}), \quad \alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s}), \mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$, $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$, and $\mathrm{Mo}(\mathrm{s})$. The results of these calculations are given in tables which can be used to assess conveniently the approximate effects on thermodynamic properties due to the differences in these temperature scales. It is found that only the most accurate thermodynamic results require examination and possible adjustment because of a change in the temperature scale.


## INTRODUCTION

The International Temperature Scale of 1990 (ITS-90) was adopted by the International Committee of Weights and Measures in 1989 (ref. 1) in accordance with the request in Resolution 7 of the 18th General Conference of Weights and Measures of 1987. This temperature scale supersedes the International Practical Temperature Scale of 1968 as amended in 1975 (IPTS-68) and the 1976 Provisional 0.5 K to 30 K Temperature Scale (EPT-76). Recent reviews (ref. 2-6) of the ITS-90 ( $T_{90}$ ) have appeared which describe how the scale is operationally defined for temperatures greater than 0.65 K in terms of vapor-pressure thermometry, gas thermometry, platinum resistance thermometry, and optical pyrometry. These reviews also present numerical and graphical comparisons between the ITS-90 and the IPTS-68 ( $T_{68}$ ) and numerical comparisons with EPT-76. In particular, the Working Group of the Comite Consultatif de Thermométrie has also prepared a monograph "Techniques for Approximating the International Temperature Scale of $1990^{\prime \prime}$ (ref. 5) which contains a discussion of the techniques used to establish and use the ITS-90. The officially established tables of differences between the ITS-90 and the EPT-76 and the IPTS-68 are given in reference 1 and are also given in references 2-6. The range of differences encountered over several temperature intervals are: -9 mK to +14 mK over the interval 15 K to 310 K ; zero to -124 mK over the interval 310 K to 930 K ; and -2.58 K to +0.36 K over the interval 930 K to 4300 K . However, it should be noted that these differences are given to an imprecision that is less than the realizable accuracy of the IPTS-68. The temperature scales preceding the most recent ones (IPTS-68, EPT-76, and ITS-90) are the IPTS-48 (also known as ITS-48), the ITS-27, and the Normal Hydrogen Scale (NHS). The latter scale was established in 1887 (ref. 7). Reference 7 contains tables giving numerical values of the approximate differences $\left(t_{27}-t_{48}\right) /^{\circ} \mathrm{C}$ as a function of $t_{27}{ }^{\circ} \mathrm{C}$ and $\left(t_{68}-t_{48}\right) /{ }^{\circ} \mathrm{C}$ as a function of $t_{68}{ }^{\circ} \mathrm{C}$.

The main effects of the ITS-90 on the field of thermodynamics are twofold. Firstly, for the most accurate work, temperatures measured with thermometers that are not recalibrated to the ITS-90 must be converted
to the new scale. Secondly, thermodynamic quantities already published in the literature and based on previous temperature scales will be changed in relation to the ITS-90. The methods for making these changes and their magnitudes need to be set out. For very many cases it is found that these corrections are much smaller than the uncertainties in the measurements and therefore can be neglected. The present article addresses these items in order to assist the thermodynamics community affected by the change in the temperature scale. Accordingly, we have: (1) constructed tables which permit one to adjust temperatures reported on the earlier temperature scales to the ITS-90; (2) summarized procedures that can be used to make the necessary adjustments in thermodynamic results; and (3) given results of some calculations of the approximate effects on thermodynamic properties due to the change from the IPTS-48 and IPTS-68 to the ITS-90.

## DIFFERENCES BETWEEN THE ITS-90 AND EARLIER TEMPERATURE SCALES

The officially recommended differences between the ITS-90 and the EPT-76 and between the ITS-90 and the IPTS-68 are given in Table VI of reference 1. This table contain the differences $\left(T_{90}-T_{76}\right) / \mathrm{K}$ and ( $\left.T_{90}-T_{68}\right) / \mathrm{K}$ as functions of $T_{90} / \mathrm{K}$ and $\left(t_{90}-t_{68}\right){ }^{\circ} \mathrm{C}$ as a function of $t_{90}{ }^{\circ} \mathrm{C}$. However, what is generally needed for the adjustment of results reported on earlier temperature scales is a table of differences as a function of the temperature on that earlier scale. Thus, Table $1\left(t_{68}{ }^{\circ} \mathrm{C}=-180\right.$ to 4000$)$ and Table $2\left(T_{68} / \mathrm{K}\right.$ $=14$ to 4300 ) in this paper have been constructed from Table VI in reference 1. In these tables the differences in the two scales are given as the quantity $\delta$ equal to either $\left(t_{90}-t_{68}\right){ }^{\circ} \mathrm{C}$ or $\left(T_{90}-T_{68}\right) / \mathrm{K}$ as a function of, respectively, $t_{68}{ }^{\circ} \mathrm{C}$ or $T_{68} / \mathrm{K} .{ }^{\dagger}$ These tables also contain values of the derivatives of the differences $\{\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})\}$ as a function of the temperature of the earlier scale. These derivatives were calculated numerically for temperatures less than $1337.15 \mathrm{~K}\left(1064^{\circ} \mathrm{C}\right)$. Here we have used the E01BEF subroutine in the NAG Library (ref. 8) to obtain a piecewise monotone cubic Hermite interpolant to the temperature differences. The derivatives are also calculated with this subroutine. Mathematical subroutines that will accomplish these types of operations are generally available (ref. 9). For temperatures higher than 1337.15 K , we used an exact equation (eq. (10) below) to obtain the derivatives. These derivatives will be needed later for the adjustment of thermodynamic properties from the IPTS-68 to the IPTS-90. Also note that there is a discontinuity in the derivative $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ at $t_{90}{ }^{\circ} \mathrm{C}=630.6$ or $T_{90} / \mathrm{K}=903.75$. In a similar manner, Table 3, which contains the differences $\delta=\left(T_{90}-T_{76}\right) / \mathrm{K}$ and derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $T_{76} / \mathrm{K}$, was constructed from Table 6 in reference 2 and Appendix A in reference 5.

Table 1.III in reference 7 contains approximate differences ( $t_{68}-t_{48}$ )/ K as a function of $t_{68}{ }^{\circ} \mathrm{C}$ from $t_{68}{ }^{\circ} \mathrm{C}$ $=-100$ to 4000 . We have used these differences and Tables 1 and 2 in this paper to construct Table 4. This table gives the differences $\delta=\left(t_{90}-t_{48}\right) /{ }^{\circ} \mathrm{C}$ and the derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $t_{48} /{ }^{\circ} \mathrm{C}$. Similarly, from Table 1.II in reference 7, which gives approximate differences $\left(t_{27}-t_{48}\right) / \mathrm{K}$ as a function of $t_{27}{ }^{\circ} \mathrm{C}$, and with Tables 1,2 , and 4 in this paper, we have constructed Table 5 . This table contains the approximate differences $\delta=\left(t_{90}-t_{27}\right) /{ }^{\circ} \mathrm{C}$ and derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $t_{27}{ }^{\circ} \mathrm{C}$. For $t_{27}{ }^{\circ} \mathrm{C}<630$, the same values given in Table 4 for $\mathrm{t}_{48}{ }^{\circ} \mathrm{C}$ should be used since the definition of the ITS-27 and the IPTS-48 are identical below that temperature (ref. 7). Although not an internationally recognized temperature scale, the National Bureau of Standards realized and maintained a scale of its own (NBS-55) for the temperature interval 13 K to 90 K from 1955 until the establishment of the IPTS-68. A table of the differences ( $T_{68}-T_{\text {NBS-55 }}$ ) as a function of $T_{\text {NBS-55 }}$ is given in Appendix C in reference 10.

[^0]The Normal Hydrogen Scale (NHS) was adopted in 1887 by the Comité International des Poids et Mesures. This scale had been developed by Chappuis and was based on gas thermometry measurements with the ice and steam points fixed at $0^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$, respectively. The initial range of this scale was $-25^{\circ} \mathrm{C}$ to $100^{\circ} \mathrm{C}$ but was gradually extended (ref. 7). If it is assumed that there were no errors in the determination of the ice and steam points and that the scale is linear over its entire range, the following relationships are obtained:

$$
\begin{align*}
& \delta=\left(t_{90}-t_{\mathrm{NHS}}\right) /{ }^{\circ} \mathrm{C}=-0.00026\left(t_{\mathrm{NHS}} /{ }^{\circ} \mathrm{C}\right)  \tag{1}\\
& d \delta / d(T / K)=-0.00026 . \tag{2}
\end{align*}
$$

To summarize, Tables $1-5$ can be used to locate conveniently both the appropriate differences and the derivatives of these differences for the conversions from the ITS-27, the IPTS-48, the IPTS-68, and the EPT-76 to the ITS-90. Equations 1 and 2 can be used to accomplish any necessary conversions from the NHS to the ITS-90. Note that on the ITS-27, $T_{27} / \mathrm{K}=t_{27}{ }^{\circ} \mathrm{C}+273.00$. On the IPTS-48, the IPTS-68, and the ITS-90, $T / \mathrm{K}=t /{ }^{\circ} \mathrm{C}+273.15$. Users of results published during the period circa $1945-1955$ should pay particular attention as to how $t /{ }^{\circ} \mathrm{C}$ was converted to $T / \mathrm{K}$ since, in many cases, the constant 273.16 was used instead of 273.15. The differences between the earlier temperature scales and ITS-90 are shown in figures 1 and 2.


Fig. 1. The quantity $\delta=\left(T_{90}-T_{x}\right) / K$ as a function of $t /{ }^{\circ} \mathrm{C}$. The two curves are: solid line, $x=68$; dashed line, $x=48$ and $x=27$.


Fig. 2. The quantity $\delta=\left(T_{90}-T_{\mathrm{x}}\right) / \mathrm{K}$ as a function of $t{ }^{\circ} \mathrm{C}$. The three curves are: solid line, $x=68$; dashed line, $x=48$; and dotted line, $x=27$.

## ANALYTICAL EQUATIONS FOR THE CONVERSION OF $\boldsymbol{T}_{68}$ TO $\boldsymbol{T}_{\mathbf{9 0}}$

Analytical equations for the differences between the ITS-90 and the IPTS-68 are also useful. They have been worked out by R.L. Rusby and have been adopted by the Comité Consultatif de Thermométrie of the Comite International des Poids et Mesures (ref. 4). The coefficients of these equations, which are now summarized, are given in Table 6.

The following polynomial represents ( $\left.T_{90}-T_{68}\right) / \mathrm{K}$ from $T_{90} / \mathrm{K}=13.8$ to 73.15 with an accuracy of about 0.001 K :

$$
\begin{equation*}
\left.\left(T_{90}-T_{68}\right) / K=a_{o}+\sum_{i=1}^{12} a_{i}\left[\left(T_{90} / K\right)-40\right) / 40\right]^{i} . \tag{3}
\end{equation*}
$$

The first derivative $\mathrm{d}\left(T_{90}-T_{68}\right) / \mathrm{d} T_{90}$ is also required for the calculation of changes to some of the
thermodynamic quantities caused by the shift in the temperature scale. This derivative is:

$$
\begin{equation*}
d\left(T_{90}-T_{68}\right) / d T_{90}=(1 / 40) \sum_{i=1}^{12} i a_{i}\left[\left(\left(T_{90} / K\right)-401 / 40\right]^{i-1} .\right. \tag{4}
\end{equation*}
$$

From 83.8 K to $903.75 \mathrm{~K}\left(-189.35^{\circ} \mathrm{C}\right.$ to $\left.630.6^{\circ} \mathrm{C}\right),\left(T_{90}-T_{68}\right) / \mathrm{K}$ is reproduced by the following equation:

$$
\begin{equation*}
\left.\left(T_{90}-T_{68}\right) / K=\sum_{i=1}^{8} b_{i}\left[\left(T_{90} / K\right)-273.15\right) / 630\right]^{i} . \tag{5}
\end{equation*}
$$

The accuracy of the above equation is 0.0015 K from 83.8 K to 273.15 K and 0.001 K from 273.15 K to 903.75 K . The derivative $\mathrm{d}\left(T_{90}-T_{68}\right) / \mathrm{d} T_{90}$ in the temperature interval 83.8 K to 903.75 K is:

$$
\begin{equation*}
d\left(T_{90}-T_{68}\right) / d T_{90}=(1 / 630) \sum_{i=1}^{8} i b_{i}\left[\left(\left(T_{90} / K\right)-273.15\right) / 630\right]^{i-1} \tag{6}
\end{equation*}
$$

It should be noted that a discontinuity in the first derivative of ( $T_{90}-T_{68}$ ) with respect to $T_{90}$ occurs at $903.75 \mathrm{~K}\left(630.6^{\circ} \mathrm{C}\right)$.

For the interval 903.75 K to $1337.33 \mathrm{~K}\left(630.6^{\circ} \mathrm{C}\right.$ to $\left.1064.18^{\circ} \mathrm{C}\right),\left(T_{90}-T_{68}\right) / \mathrm{K}$ is reproduced to within about 0.01 K by:

$$
\begin{equation*}
\left(T_{90}-T_{68}\right) / K=c_{O}+\sum_{i=1}^{7} c_{i}\left\{\left(\left\{T_{90} / K\right)-1173.15\right\rangle / 300\right]^{i} \tag{7}
\end{equation*}
$$

The first derivative in this temperature interval is:

$$
\begin{equation*}
d\left(T_{90}-T_{68}\right) / d T_{90}=(1 / 300) \sum_{i=1}^{7} i c_{i}\left[\left(\left(T_{90} / K\right)-1173.15\right) / 300\right]^{i-1} . \tag{8}
\end{equation*}
$$

Above $1337.33 \mathrm{~K}\left(1064.18^{\circ} \mathrm{C}\right)$, an operational equation for $\left(T_{90}-T_{68}\right) / \mathrm{K}$ is:

$$
\begin{equation*}
\left.\left(T_{90}-T_{68}\right) / K=\left(T_{90}^{2} / K\right)\left[\left\{T_{90}(A u)-T_{68}(A u)\right) / / T_{90}(A u) T_{68}(A u)\right]\right] \tag{9}
\end{equation*}
$$

where the freezing temperature of gold $T_{90}(\mathrm{Au})=1337.33 \mathrm{~K}$ and $T_{68}(\mathrm{Au})=1337.58 \mathrm{~K}$. Thus, the term in square brackets is a constant equal to $-1.398 \times 10^{-7}$. This equation is a simpler and essentially equivalent form of eq. (1.5) in reference 4. The silver, gold, and copper freezing temperatures can all be used in the construction of a pyrometric scale. The derivative obtained from the above equation is:

$$
\begin{equation*}
\left.d\left(T_{90}-T_{68}\right) / d T_{90}=2 T_{90}\left[\left[T_{90}(A u)-T_{68}(A u)\right) / / T_{90}(A u) T_{68}(A u)\right)\right] . \tag{10}
\end{equation*}
$$

Eqs. (5) and (7) above have been changed, respectively, from eqs. (1.3) and (1.4) in Table 1.6 in reference 4 to use temperature in kelvins rather than Celsius.

Rusby has recently (ref. 11) modified the above equations so as to give directly ( $T_{90}-T_{68}$ ) from $T_{68}$. The temperature intervals over which Rusby's modified equations are valid are also slightly different: 13.81 K to 83.8 K for eq. (3); 73.15 K to 903.89 K for eq. (5); and 903.89 K to 1337.58 K for eq. (7). The derivatives which are calculated with the above equations are in reasonable agreement with those which are directly calculated with a Hermite interpolant from the officially (ref. 1) established differences $\left(T_{90}-T_{68}\right) / \mathrm{K}$. However, the derivatives calculated from the above equations are smoother than those
obtained from the Hermite interpolant. As is shown below, the effects on thermodynamic properties due to the change from IPTS-68 to the ITS-90 are almost always within the experimental errors of the property measurements. Thus, it is highly unlikely that the uncertainties in the derivatives $\mathrm{d}\left(T_{90}-T_{68}\right) / \mathrm{d} T_{90}$ will have a substantive effect on the calculated adjustments. If, however, the actual property measurements should be of exceptional accuracy, it is recommended that such results be recalculated with a point to point conversion of each temperature to the ITS-90.

## CONVERSION OF THERMODYNAMIC QUANTITIES

For the most accurate thermodynamic results, it is preferable to change the temperature of the original work to $T_{90}$ as outlined above and then convert the thermodynamic quantities to this basis. However, equations for correcting thermodynamic quantities directly due to the shift in temperature scale have been derived by Douglas (ref. 12). Taylor expansions of the enthalpy $H$, heat capacity $C_{p}$, and entropy $S$ result in exact equations in infinite series from which the corrections $\delta H, \delta C_{p}$, or $\delta S$ are determined. The differences between the temperatures of the ITS-90 and the IPTS-68 are sufficiently small to justify dropping higher order terms in these infinite series to produce simpler but approximate equations with sufficient accuracy for nearly all cases, one exception being in the region of sharp transitions. The simplified equations for the changes in the enthalpy, heat capacity, and entropy are, respectively:

$$
\begin{align*}
& \delta H=-\left(T_{90}-T_{x}\right) C_{p}  \tag{11}\\
& \delta C_{p}=-\left(T_{90}-T_{x}\right) d C_{p} / d T-C_{p} d\left(T_{90}-T_{x}\right) / d T  \tag{12}\\
& \left.\delta S=-\int_{0}^{T}\left(T_{90}-T_{x}\right) C_{p} / T^{2}\right\} d T-\left(T_{90}-T_{x}\right) C_{p} / T .
\end{align*}
$$

Here, $T_{\mathrm{x}}$ is the thermodynamic temperature on any of the earlier scales (NHS, ITS-27, IPTS-48, IPTS-68, or EPT-76). Eqs. (11), (12), and (13) correspond, respectively, to eqs. (52), (53), and (56) of reference 12. All properties are those at the nominal value of $T_{90}$ where the corrected property is desired. Errors in the differences in calculated thermodynamic properties (e.g. $C_{p}(90)-C_{p}(68)$ ) may also arise from uncertainties in the various terms (particularly $\mathrm{d} C_{p} / \mathrm{d} T$ and $\mathrm{d}\left(T_{90}-T_{\mathrm{x}}\right) / \mathrm{d} T$ which are used in eq. (12). Also, we have sometimes used previously fitted results to obtain the approximate differences in thermodynamic properties given in Table 7. We note that the fitting process tends to produce smooth results and, in doing this, it tends to minimize the effects of errors in the measurements, including any errors in the thermometry and in the temperature scale used in the study. Thus, if one wishes to obtain accurate values of the thermodynamic properties on the ITS-90, it is preferable to correct the original results to ITS-90 and then to perform the desired fit to the results rather than to use the Douglas equations to adjust results which have already been fit. Therefore, while the differences in thermodynamic properties obtained with the Douglas equations are satisfactory for the purpose of assessing the approximate effects on thermodynamic properties due to differences in temperature scales, the fitted results of a study based upon an earlier temperature scale may have already smoothed out some of the experimental detail in the original measurements. Thus, while exact results will not be obtained with eqs. (11), (12), and (13), they should suffice for the calculation of approximate differences in thermodynamic properties.

Accordingly, we have used eqs. (11), (12), and (13) to assess the effects of the changes from the ITS-68 to the ITS-90 on the thermodynamic properties for six materials which were selected to cover a wide range of temperature. These materials include the heat-capacity studies on $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ by adiabatic calorimetry (ref. 13), $\mathrm{BaSnF}_{4}$ by both adiabatic calorimetry and differential scanning calorimetry (ref. 14, 15), $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ (ref. 16, 17, 18), $\mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ (ref. 19, 20), and $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ (ref. 21, 22) by adiabatic and drop-calorimetry, and $\mathrm{Mo}(\mathrm{s})$ by adiabatic, drop, and pulse-heating calorimetry (ref. 23, 24). The results of these calculations are given in Table 7 for the change from IPTS-68 to ITS-90 and in Table 8 for the
change from IPTS-48 to ITS-90. The predominant contribution made by the term $C_{p} \mathrm{~d}\left(T_{90}-T_{\mathrm{x}}\right) / \mathrm{d} T$ in eq. (12) leads one to predict that the effects on $10^{2}\left\{C_{p}(90)-C_{p}(68)\right\} / C_{p}(90)$ are the greatest at temperatures between 16 K and 18 K and at temperatures between 903.75 K and 1320 K . For this reason results are given in Tables 7 and 8 at appropriately spaced temperature intervals. Thus, these two tables can be used to assess the approximate effects on thermodynamic properties due to the changes in these temperature scales (IPTS-48 and IPTS-68) to ITS-90. If the inaccuracies in the results are judged to be less than the approximate effects given in Table 7 (or Table 8), then a recalculation of the original results is clearly justified. In some cases the imprecisions in the results used to construct Tables 7 and 8 are less than the approximate differences given for the temperature interval 903.75 K to 1320 K (Table 7 and Table 8) and for the temperature interval 100 K to 400 K (Table 8). Indeed, some investigators have noted that problems in the temperature scale could account for some of the deviations in their results. For example, Ditmars and Douglas (ref. 18) found that the fit of their experimental results was not smooth in the interval $T=873 \mathrm{~K}$ to $T=1023 \mathrm{~K}$. They wrote (ref. 18) that the "more likely contributing causes to the nonsmoothness (in both the present and the 1956 NBS enthalpy data for $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}$ ) are possible systematic error in the realization of the International Practical Temperature Scale of 1968 in these measurements and especially the differences between this scale and the true thermodynamic one." Similarly, Furukawa and Reilly (ref. 25, 26) were able to detect "anomalies" in the heat capacities of several substances in the region of the oxygen point where the temperature scale (IPTS-48) was based on the joining of the results obtained from platinum resistance and thermocouple thermometers. These anomalies were readily removed after the temperature scale had been properly adjusted. Ginnings states (ref. 27) that "in the range $0-100^{\circ} \mathrm{C}$, the specific heat of liquid water is believed to be known within $0.01-0.02$ per cent". Thus, the results of Osborne et al. (ref. 28) on the heat capacity of water should be examined in light of the changes in the temperature scale. The results of Ditmars and Douglas (ref. 18, 20, 22), those of Furukawa and Saba (ref. 19, 21), those of Chang (17), and those of Osborne et al. (ref. 28) have a precision greater than the vast majority of thermodynamic measurements in the literature. Thus, only results which are judged to have inaccuracies less than those effects which are calculated with eqs. (11), (12), or (13) above require examination and possible adjustment because of the change(s) in the temperature scale. Thermodynamic properties which are calculated from statistical mechanics are implicitly done with thermodynamic temperatures. Therefore, no adjustments of the types discussed in this paper are required in this case.

## ADDENDUM

Eight laboratories working under the auspices of Working Group 2 of the Comité Consultatif de Thermométrie have recently determined (ref. 29, 30,31) the differences $\delta=\left(t_{90}-t_{68}\right) /{ }^{\circ} \mathrm{C}$ from $630^{\circ} \mathrm{C}$ to $1064^{\circ} \mathrm{C}$ with temperatures on the International Temperature Scale of 1990 determined with platinum resistance thermometers. These results give a set of temperature differences which differ from the earlier results (see ref. 1 and Tables 1 and 2 in this paper) by as much as 0.32 K over this temperature interval. Burns et al. (ref. 30) have represented these new differences in terms of the following polynomial:

$$
\begin{align*}
\delta= & 7.8687209 \times 10^{1}-4.7135991 \times 10^{-1}\left(t_{90}{ }^{\circ} \mathrm{C}\right)+1.0954715 \times 10^{-3}\left(t_{9} 0^{\circ} \mathrm{C}\right)^{2} \\
& -1.2357884 \times 10^{-6}\left(t_{90}{ }^{\circ} \mathrm{C}\right)^{3}+6.7736583 \times 10^{-10}\left(t_{90}{ }^{\circ} \mathrm{C}\right)^{4}-1.4458081 \times 10^{-13}\left(t_{90}{ }^{\circ} \mathrm{C}\right)^{5} . \tag{14}
\end{align*}
$$

From $680^{\circ} \mathrm{C}$ to $880^{\circ} \mathrm{C}(T=953.15 \mathrm{~K}$ to $T=1153.15 \mathrm{~K})$ the absolute values of these new temperature differences and the absolute values of the temperature derivatives of these differences are substantially less than the corresponding quantities given in Tables 1 and 2 . Thus, the approximate differences in thermodynamic properties calculated with these new temperature differences and the new temperature derivatives of these differences will be substantially less for the temperature interval 953 K to 1153 K than the approximate differences given in Tables 7 and 8.

TABLE 1. Differences $\delta=\left(t_{90}-t_{68}\right){ }^{\circ} \mathrm{C}$ and derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $t_{68}{ }^{\circ} \mathrm{C}$.

| $\mathrm{t}_{68}{ }^{\circ} \mathrm{C}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ | $t_{68}{ }^{\circ} \mathrm{C}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ | $t_{68}{ }^{\circ} \mathrm{C}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -180 | 0.008 | 0.00020 | 400 | -0.048 | -0.00024 | 980 | -0.17 | -0.0010 |
| -170 | 0.010 | 0.00020 | 410 | -0.051 | -0.00024 | 990 | -0.18 | -0.0010 |
| -160 | 0.012 | 0.00013 | 420 | -0.053 | -0.00024 | 1000 | -0.19 | -0.0010 |
| -150 | 0.013 | 0.00010 | 430 | -0.056 | -0.00030 | 1010 | -0.20 | -0.0010 |
| -140 | 0.014 | 0.00000 | 440 | -0.059 | -0.00030 | 1020 | -0.21 | -0.0010 |
| -130 | 0.014 | 0.00000 | 450 | -0.062 | -0.00030 | 1030 | -0.22 | -0.0010 |
| -120 | 0.014 | 0.00000 | 460 | -0.065 | -0.00030 | 1040 | -0.23 | -0.0010 |
| -110 | 0.013 | 0.00000 | 470 | -0.068 | -0.00034 | 1050 | -0.24 | -0.0010 |
| -100 | 0.013 | 0.00000 | 480 | -0.072 | -0.00034 | 1060 | -0.25 | 0.0000 |
| -90 | 0.012 | 0.00000 | 490 | -0.075 | -0.00034 | 1070 | -0.25 | -0.00038 |
| -80 | 0.012 | 0.00000 | 500 | -0.079 | -0.00040 | 1100 | -0.26 | 0.00038 |
| -70 | 0.011 | -0.00010 | 510 | -0.083 | -0.00040 | 1200 | -0.30 | -0.00041 |
| -60 | 0.010 | -0.00010 | 520 | -0.087 | -0.00034 | 1300 | -0.35 | -0.00044 |
| -50 | 0.009 | -0.00010 | 530 | -0.090 | -0.00034 | 1400 | -0.39 | -0.00047 |
| -40 | 0.008 | -0.00013 | 540 | -0.094 | -0.00040 | 1500 | -0.44 | -0.00050 |
| -30 | 0.006 | -0.00020 | 550 | -0.098 | -0.00034 | 1600 | -0.49 | -0.00052 |
| -20 | 0.004 | -0.00020 | 560 | -0.101 | -0.00034 | 1700 | -0.54 | -0.00055 |
| -10 | 0.002 | -0.00020 | 570 | -0.105 | -0.00034 | 1800 | -0.60 | -0.00058 |
| 0 | 0.000 | -0.00020 | 580 | -0.108 | -0.00034 | 1900 | -0.66 | -0.00061 |
| 10 | -0.002 | -0.00024 | 590 | -0.112 | -0.00034 | 2000 | -0.72 | -0.00064 |
| 20 | -0.005 | -0.00024 | 600 | -0.115 | -0.00030 | 2100 | -0.79 | -0.00066 |
| 30 | -0.007 | -0.00024 | 610 | -0.118 | -0.00034 | 2200 | -0.85 | -0.00069 |
| 40 | -0.010 | -0.00030 | 620 | -0.122 | -0.00034 | 2300 | -0.93 | -0.00072 |
| 50 | -0.013 | -0.00030 | 630 | -0.125 | 0.00000 | 2400 | -1.00 | -0.00075 |
| 60 | -0.016 | -0.00024 | 640 | -0.08 | 0.0049 | 2500 | -1.07 | -0.00078 |
| 70 | -0.018 | -0.00024 | 650 | -0.03 | 0.0050 | 2600 | -1.15 | -0.00080 |
| 80 | -0.021 | -0.00030 | 660 | 0.02 | 0.0044 | 2700 | -1.24 | -0.00083 |
| 90 | -0.024 | -0.00024 | 670 | 0.06 | 0.0044 | 2800 | -1.32 | -0.00086 |
| 100 | -0.026 | -0.00020 | 680 | 0.11 | 0.0050 | 2900 | -1.41 | -0.00089 |
| 110 | -0.028 | -0.00020 | 690 | 0.16 | 0.0044 | 3000 | -1.50 | -0.00091 |
| 120 | -0.030 | -0.00020 | 700 | 0.20 | 0.0040 | 3100 | -1.59 | -0.00094 |
| 130 | -0.032 | -0,00020 | 710 | 0.24 | 0.0040 | 3200 | -1.69 | -0.00097 |
| 140 | -0.034 | -0.00020 | 720 | 0.28 | 0.0034 | 3300 | -1.78 | -0.00100 |
| 150 | -0.036 | -0.00013 | 730 | 0.31 | 0.0024 | 3400 | -1.89 | -0.00103 |
| 160 | -0.037 | -0.00010 | 740 | 0.33 | 0.0020 | 3500 | -1.99 | -0.00105 |
| 170 | -0.038 | -0.00010 | 750 | 0.35 | 0.0013 | 3600 | -2.10 | -0.00108 |
| 180 | -0.039 | 0.00000 | 760 | 0.36 | 0.0000 | 3700 | -2.21 | -0.00111 |
| 190 | -0.039 | 0.00000 | 770 | 0.36 | 0.0000 | 3800 | -2.32 | -0.00114 |
| 200 | -0.040 | 0.00000 | 780 | 0.36 | 0.0000 | 3900 | -2.43 | -0.00117 |
| 210 | -0.040 | 0.00000 | 790 | 0.35 | -0.0010 | 4000 | -2.55 | -0.00119 |
| 220 | -0.040 | 0.00000 | 800 | 0.34 | -0.0013 |  |  |  |
| 230 | -0.040 | 0.00000 | 810 | 0.32 | -0.0024 |  |  |  |
| 240 | -0.040 | 0.00000 | 820 | 0.29 | -0.0034 |  |  |  |
| 250 | -0.040 | 0.00000 | 830 | 0.25 | -0.0034 |  |  |  |
| 260 | -0.040 | 0.00000 | 840 | 0.22 | -0.0034 |  |  |  |
| 270 | -0.039 | 0.00000 | 850 | 0.18 | -0.0040 |  |  |  |
| 280 | -0.039 | 0.00000 | 860 | 0.14 | -0.0040 |  |  |  |
| 290 | -0.039 | 0.00000 | 870 | 0.10 | -0.0040 |  |  |  |
| 300 | -0.039 | 0.00000 | 880 | 0.06 | -0.0034 |  |  |  |
| 310 | -0.039 | 0.00000 | 890 | 0.03 | -0.0034 |  |  |  |
| 320 | -0.039 | 0.00000 | 900 | -0.01 | -0.0027 |  |  |  |
| 330 | -0.040 | 0.00000 | 910 | -0.03 | -0.0024 |  |  |  |
| 340 | -0.040 | 0.00000 | 920 | -0.06 | -0.0024 |  |  |  |
| 350 | -0.041 | -0.00010 | 930 | -0.08 | -0.0020 |  |  |  |
| 360 | -0.042 | -0.00010 | 940 | -0.10 | -0.0020 |  |  |  |
| 370 | -0.043 | -0.00013 | 950 | -0.12 | -0.0020 |  |  |  |
| 380 | -0.045 | -0.00013 | 960 | -0.14 | -0.0020 |  |  |  |
| 390 | -0.046 | -0.00013 | 970 | -0.16 | -0.0013 |  |  |  |

TABLE 2. Differences $\delta=\left(T_{90}-T_{68}\right) / \mathrm{K}$ and derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $T_{68} / \mathrm{K}$.

| $T_{68} / \mathrm{K}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ | $T_{68} / \mathrm{K}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ | $T_{68} / \mathrm{K}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14 | -0.006 | 0.0050 | 74 | 0.007 | 0.0000 | 430 | -0.037 | -0.00013 |
| 15 | -0.003 | 0.0000 | 75 | 0.008 | 0.0000 | 440 | -0.038 | -0.00010 |
| 16 | -0.004 | -0.0013 | 76 | 0.008 | 0.0000 | 450 | -0.039 | 0.00000 |
| 17 | -0.006 | -0.0020 | 77 | 0.008 | 0.0000 | 460 | -0.039 | 0.00000 |
| 18 | -0.008 | -0.0013 | 78 | 0.008 | 0.0000 | 470 | -0.040 | 0.00000 |
| 19 | -0.009 | 0.0000 | 79 | 0.008 | 0.0000 | 480 | -0.040 | 0.00000 |
| 20 | -0.009 | 0.0000 | 80 | 0.008 | 0.0000 | 490 | -0.040 | 0.00000 |
| 21 | -0.008 | 0.0010 | 81 | 0.008 | 0.0000 | 500 | -0.040 | 0.00000 |
| 22 | -0.007 | 0.0000 | 82 | 0.008 | 0.0000 | 510 | -0.040 | 0.00000 |
| 23 | -0.007 | 0.0000 | 83 | 0.008 | 0.0000 | 520 | -0.040 | 0.00000 |
| 24 | -0.006 | 0.0010 | 84 | 0.008 | 0.0000 | 530 | -0.040 | 0.00000 |
| 25 | -0.005 | 0.0010 | 85 | 0.008 | 0.0000 | 540 | -0.039 | 0.00000 |
| 26 | -0.004 | 0.0000 | 86 | 0.008 | 0.0000 | 550 | -0.039 | 0.00000 |
| 27 | -0.004 | 0.0000 | 87 | 0.008 | 0.0000 | 560 | -0.039 | 0.00000 |
| 28 | -0.005 | -0.0010 | 88 | 0.008 | 0.0000 | 570 | -0.039 | 0.00000 |
| 29 | -0.006 | 0.0000 | 89 | 0.008 | 0.0000 | 580 | -0.039 | 0.00000 |
| 30 | -0.006 | 0.0000 | 90 | 0.008 | 0.0000 | 590 | -0.039 | 0.00000 |
| 31 | -0.007 | -0.0010 | 91 | 0.008 | 0.0000 | 600 | -0.040 | 0.00000 |
| 32 | -0.008 | 0.0000 | 92 | 0.008 | 0.0000 | 610 | -0.040 | 0.00000 |
| 33 | -0.008 | 0.0000 | 93 | 0.008 | 0.0000 | 620 | -0.041 | -0.00010 |
| 34 | -0.008 | 0.0000 | 94 | 0.008 | 0.0000 | 630 | -0.042 | -0.00010 |
| 35 | -0.007 | 0.0000 | 95 | 0.008 | 0.0000 | 640 | -0.043 | -0.00010 |
| 36 | -0.007 | 0.0000 | 96 | 0.008 | 0.0000 | 650 | -0.044 | -0.00013 |
| 37 | -0.007 | 0.0000 | 97 | 0.009 | 0.0000 | 660 | -0.046 | -0.00013 |
| 38 | -0.006 | 0.0000 | 98 | 0.009 | 0.0000 | 670 | -0.047 | -0.00015 |
| 39 | -0.006 | 0.0000 | 99 | 0.009 | 0.0000 | 680 | -0.050 | -0.00024 |
| 40 | -0.006 | 0.0000 | 100 | 0.009 | 0.0000 | 690 | -0.052 | -0.00024 |
| 41 | -0.006 | 0.0000 | 110 | 0.011 | 0.00020 | 700 | -0.055 | -0.00030 |
| 42 | -0.006 | 0.0000 | 120 | 0.013 | 0.00013 | 710 | -0.058 | -0.00030 |
| 43 | -0.006 | 0.0000 | 130 | 0.014 | 0.00000 | 720 | -0.061 | -0.00030 |
| 44 | -0.006 | 0.0000 | 140 | 0.014 | 0.00000 | 730 | -0.064 | -0.00030 |
| 45 | -0.007 | 0.0000 | 150 | 0.014 | 0.00000 | 740 | -0.067 | -0.00034 |
| 46 | -0.007 | 0.0000 | 160 | 0.014 | 0.00000 | 750 | -0.071 | -0.00034 |
| 47 | -0.007 | 0.0000 | 170 | 0.013 | -0.00010 | 760 | -0.074 | -0.00034 |
| 48 | -0.006 | 0.0000 | 180 | 0.012 | 0.00000 | 770 | -0.078 | -0.00040 |
| 49 | -0.006 | 0.0000 | 190 | 0.012 | 0.00000 | 780 | -0.082 | -0.00040 |
| 50 | -0.006 | 0.0000 | 200 | 0.011 | -0.00010 | 790 | -0.086 | -0.00034 |
| 51 | -0.005 | 0.0000 | 210 | 0.010 | -0.00010 | 800 | -0.089 | -0.00034 |
| 52 | -0.005 | 0.0000 | 220 | 0.009 | -0.00010 | 810 | -0.093 | -0.00040 |
| 53 | -0.004 | 0.0010 | 230 | 0.008 | -0.00010 | 820 | -0.097 | -0.00034 |
| 54 | -0.003 | 0.0010 | 240 | 0.007 | -0.00013 | 830 | -0.100 | -0.00034 |
| 55 | -0.002 | 0.0010 | 250 | 0.005 | -0.00020 | 840 | -0.104 | -0.00034 |
| 56 | -0.001 | 0.0010 | 260 | 0.003 | -0.00020 | 850 | -0.107 | -0.00034 |
| 57 | 0.000 | 0.0010 | 270 | 0.001 | -0.00026 | 860 | -0.111 | -0.00034 |
| 58 | 0.001 | 0.0010 | 273.15 | 0.000 | -0.00021 | 870 | -0.114 | -0.00030 |
| 59 | 0.002 | 0.0010 | 280 | -0.001 | -0.00019 | 880 | -0.117 | -0.00034 |
| 60 | 0.003 | 0.0000 | 290 | -0.004 | -0.00024 | 890 | -0.121 | -0.00034 |
| 61 | 0.003 | 0.0000 | 300 | -0.006 | -0.00024 | 900 | -0.124 | 0.00000 |
| 62 | 0.004 | 0.0000 | 310 | -0.009 | -0.00030 | 903.75 | -0.125 | - |
| 63 | 0.004 | 0.0000 | 320 | -0.012 | -0.00030 | 910 | -0.09 | 0.0037 |
| 64 | 0.005 | 0.0000 | 330 | -0.015 | -0.00024 | 920 | -0.05 | 0.0044 |
| 65 | 0.005 | 0.0000 | 340 | -0.017 | -0.00024 | 930 | 0.00 | 0.0050 |
| 66 | 0.006 | 0.0000 | 350 | -0.020 | -0.00030 | 940 | 0.05 | 0.0044 |
| 67 | 0.006 | 0.0000 | 360 | -0.023 | -0.00024 | 950 | 0.09 | 0.0044 |
| 68 | 0.007 | 0.0000 | 370 | -0.025 | -0.00020 | 960 | 0.14 | 0.0050 |
| 69 | 0.007 | 0.0000 | 380 | -0.027 | -0.00020 | 970 | 0.19 | 0.0044 |
| 70 | 0.007 | 0.0000 | 390 | -0.029 | -0.00020 | 980 | 0.23 | 0.0040 |
| 71 | 0.007 | 0.0000 | 400 | -0.031 | -0.00020 | 990 | 0.27 | 0.0034 |
| 72 | 0.007 | 0.0000 | 410 | -0.033 | -0.00020 | 1000 | 0.30 | 0.0024 |
| 73 | 0.007 | 0.0000 | 420 | -0.035 | -0.00020 | 1010 | 0.32 | 0.0020 |

TABLE 2 (continued)

| $T_{68} / \mathrm{K}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ | $T_{68} / \mathrm{K}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ | $T_{68} / \mathrm{K}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1020 | 0.34 | 0.0020 | 1300 | -0.22 | -0.0010 | 3500 | -1.71 | -0.00098 |
| 1030 | 0.36 | 0.0000 | 1310 | -0.23 | -0.0010 | 3600 | -1.81 | -0.00101 |
| 1040 | 0.36 | 0.0000 | 1320 | -0.24 | -0.0010 | 3700 | -1.92 | -0.00103 |
| 1050 | 0.36 | 0.0000 | 1330 | -0.25 | -0.00030 | 3800 | -2.02 | -0.00106 |
| 1060 | 0.35 | -0.0010 | 1340 | -0.25 | -0.00037 | 3900 | -2.13 | -0.00109 |
| 1070 | 0.34 | -0.0010 | 1350 | -0.26 | -0.00038 | 4000 | -2.24 | -0.00112 |
| 1080 | 0.33 | -0.0015 | 1360 | -0.26 | -0.00038 | 4100 | -2.35 | -0.00115 |
| 1090 | 0.30 | -0.0034 | 1400 | -0.27 | -0.00039 | 4200 | -2.46 | -0.00117 |
| 1100 | 0.26 | -0.0034 | 1500 | -0.31 | -0.00042 | 4300 | -2.58 | -0.00120 |
| 1110 | 0.23 | -0.0034 | 1600 | -0.36 | -0.00045 |  |  |  |
| 1120 | 0.19 | -0.0040 | 1700 | -0.40 | -0.00048 |  |  |  |
| 1130 | 0.15 | -0.0040 | 1800 | -0.45 | -0.00050 |  |  |  |
| 1140 | 0.11 | -0.0040 | 1900 | -0.50 | -0.00053 |  |  |  |
| 1150 | 0.07 | -0.0034 | 2000 | -0.56 | -0.00056 |  |  |  |
| 1160 | 0.04 | -0.0034 | 2100 | -0.62 | -0.00059 |  |  |  |
| 1170 | 0.00 | -0.0027 | 2200 | -0.68 | -0.00061 |  |  |  |
| 1180 | -0.02 | -0.0024 | 2300 | -0.74 | -0.00064 |  |  |  |
| 1190 | -0.05 | -0.0024 | 2400 | -0.81 | -0.00067 |  |  |  |
| 1200 | -0.07 | -0.0020 | 2500 | -0.87 | -0.00070 |  |  |  |
| 1210 | -0.09 | -0.0020 | 2600 | -0.95 | -0.00073 |  |  |  |
| 1220 | -0.11 | -0.0020 | 2700 | -1.02 | -0.00075 |  |  |  |
| 1230 | -0.13 | -0.0020 | 2800 | -1.09 | -0.00078 |  |  |  |
| 1240 | -0.15 | -0.0020 | 2900 | -1.17 | -0.00081 |  |  |  |
| 1250 | -0.17 | -0.0013 | 3000 | -1.26 | -0.00084 |  |  |  |
| 1260 | -0.18 | -0.0010 | 3100 | -1.34 | -0.00087 |  |  |  |
| 1270 | -0.19 | -0.0010 | 3200 | -1.43 | -0.00089 |  |  |  |
| 1280 | -0.20 | -0.0010 | 3300 | -1.52 | -0.00092 |  |  |  |
| 1290 | -0.21 | -0.0010 | 3400 | -1.62 | -0.00095 |  |  |  |

TABLE 3. Differences $\delta=\left(T_{90}-T_{76}\right) / \mathrm{K}$ and derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $T_{76} / \mathrm{K}$.

|  |  |  |
| :---: | :---: | :---: |
| $T_{76} / \mathrm{K}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ |
|  |  | -0.00010 |
| 5 | -0.0001 | -0.00010 |
| 6 | -0.0002 | -0.00010 |
| 7 | -0.0003 | -0.00010 |
| 8 | -0.0004 | -0.00010 |
| 9 | -0.0005 | -0.00010 |
| 10 | -0.0006 | -0.00010 |
| 11 | -0.0007 | -0.00013 |
| 12 | -0.0008 | -0.00013 |
| 13 | -0.0010 | -0.00013 |
| 14 | -0.0011 | -0.00013 |
| 15 | -0.0013 | -0.00013 |
| 16 | -0.0014 | -0.00020 |
| 17 | -0.0016 | -0.00020 |
| 18 | -0.0018 | -0.00020 |
| 19 | -0.0020 | -0.00024 |
| 20 | -0.0022 | -0.00024 |
| 21 | -0.0025 | -0.00024 |
| 22 | -0.0027 | -0.00024 |
| 23 | -0.0030 | -0.00024 |
| 24 | -0.0032 | -0.00030 |
| 25 | -0.0035 | -0.00030 |
| 26 | -0.0038 | -0.00030 |

TABLE 4. Approximate differences $\left.\delta=\left(t_{90}-t_{48}\right)\right)^{\circ} \mathrm{C}$ and derivatives of these differences $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ as a function of $t_{48}{ }^{\circ} \mathrm{C}$.

| $t_{48}{ }^{\circ} \mathrm{C}$ | $\delta$ | d $\delta / \mathrm{d}(T / \mathrm{K})$ | $t_{48}{ }^{\circ} \mathrm{C}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ | $t_{48}{ }^{\circ} \mathrm{C}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -180 | 0.020 | 0.00000 | 400 | 0.028 | -0.00027 | 980 | 1.01 | 0.0020 |
| -170 | 0.017 | -0.00046 | 410 | 0.024 | -0.00027 | 990 | 1.03 | 0.0020 |
| -160 | 0.007 | -0.00082 | 420 | 0.022 | -0.00024 | 1000 | 1.05 | 0.0020 |
| -150 | 0.000 | 0.00000 | 430 | 0.019 | -0.00034 | 1010 | 1.07 | 0.0020 |
| -140 | 0.001 | 0.00018 | 440 | 0.015 | -0.00034 | 1020 | 1.09 | 0.0020 |
| -130 | 0.008 | 0.00079 | 450 | 0.012 | -0.00030 | 1030 | 1.11 | 0.0020 |
| -120 | 0.017 | 0.00090 | 460 | 0.009 | -0.00024 | 1040 | 1.13 | 0.0020 |
| -110 | 0.026 | 0.00090 | 470 | 0.007 | -0.00024 | 1050 | 1.15 | 0.0020 |
| -100 | 0.035 | 0.00072 | 480 | 0.004 | -0.00024 | 1060 | 1.17 | 0.0020 |
| -90 | 0.041 | 0.00048 | 490 | 0.002 | -0.00020 | 1070 | 1.19 | 0.0018 |
| -80 | 0.045 | 0.00000 | 500 | 0.000 | -0.00013 | 1100 | 1.2 | 0.0016 |
| -70 | 0.045 | 0.00000 | 510 | -0.001 | -0.00010 | 1200 | 1.4 | 0.00076 |
| -60 | 0.042 | -0.00034 | 520 | -0.002 | 0.00000 | 1300 | 1.5 | 0.00076 |
| -50 | 0.038 | -0.00048 | 530 | -0.001 | 0.00010 | 1400 | 1.6 | 0.0015 |
| -40 | 0.032 | -0.00069 | 540 | 0.000 | 0.00013 | 1500 | 1.8 | 0.0015 |
| -30 | 0.024 | -0.00080 | 550 | 0.002 | 0.00029 | 1600 | 1.9 | 0.0015 |
| -20 | 0.016 | -0.00080 | 560 | 0.007 | 0.00044 | 1700 | 2.1 | 0.0014 |
| -10 | 0.008 | -0.00080 | 570 | 0.011 | 0.00051 | 1800 | 2.2 | 0.0014 |
| 0 | 0.000 | -0.00069 | 580 | 0.018 | 0.00070 | 1900 | 2.3 | 0.0014 |
| 10 | -0.006 | -0.00060 | 590 | 0.025 | 0.00082 | 2000 | 2.5 | 0.0017 |
| 20 | -0.012 | -0.00048 | 600 | 0.035 | 0.0011 | 2100 | 2.7 | 0.0017 |
| 30 | -0.016 | -0.00040 | 610 | 0.047 | 0.0012 | 2200 | 2.9 | 0.0017 |
| 40 | -0.020 | -0.00034 | 620 | 0.060 | 0.0014 | 2300 | 3.1 | 0.0016 |
| 50 | -0.023 | -0.00030 | 630 | 0.075 | 0.0025 | 2400 | 3.2 | 0.0017 |
| 60 | -0.026 | 0.00000 | 640 | 0.15 | 0.0072 | 2500 | 3.4 | 0.0022 |
| 70 | -0.026 | 0.00000 | 650 | 0.22 | 0.0075 | 2600 | 3.7 | 0.0015 |
| 80 | -0.027 | 0.00000 | 660 | 0.30 | 0.0075 | 2700 | 3.8 | 0.0015 |
| 90 | -0.027 | 0.00000 | 670 | 0.37 | 0.0075 | 2800 | 4.0 | 0.0021 |
| 100 | -0.026 | 0.00013 | 680 | 0.45 | 0.0075 | 2900 | 4.2 | 0.0021 |
| 110 | -0.024 | 0.00013 | 690 | 0.52 | 0.0070 | 3000 | 4.4 | 0.0021 |
| 120 | -0.023 | 0.00015 | 700 | 0.59 | 0.0070 | 3100 | 4.6 | 0.0020 |
| 130 | -0.020 | 0.00024 | 710 | 0.66 | 0.0070 | 3200 | 4.8 | 0.0024 |
| 140 | -0.018 | 0.00020 | 720 | 0.73 | 0.0058 | 3300 | 5.1 | 0.0024 |
| 150 | -0.016 | 0.00027 | 730 | 0.78 | 0.0050 | 3400 | 5.3 | 0.0019 |
| 160 | -0.012 | 0.00034 | 740 | 0.83 | 0.0050 | 3500 | 5.5 | 0.0024 |
| 170 | -0.009 | 0.00034 | 750 | 0.88 | 0.0044 | 3600 | 5.8 | 0.0023 |
| 180 | -0.005 | 0.00040 | 760 | 0.92 | 0.0027 | 3700 | 6.0 | 0.0023 |
| 190 | -0.001 | 0.00040 | 770 | 0.94 | 0.0024 | 3800 | 6.3 | 0.0029 |
| 200 | 0.003 | 0.00040 | 780 | 0.97 | 0.0024 | 3900 | 6.6 | 0.0022 |
| 210 | 0.007 | 0.00040 | 790 | 0.99 | 0.0020 | 4000 | 6.8 | 0.0013 |
| 220 | 0.011 | 0.00034 | 800 | 1.01 | 0.0013 |  |  |  |
| 230 | 0.014 | 0.00034 | 810 | 1.02 | 0.0000 |  |  |  |
| 240 | 0.018 | 0.00034 | 820 | 1.01 | -0.0010 |  |  |  |
| 250 | 0.021 | 0.00030 | 830 | 1.00 | 0.0000 |  |  |  |
| 260 | 0.024 | 0.00034 | 840 | 1.00 | 0.0000 |  |  |  |
| 270 | 0.028 | 0.00027 | 850 | 0.99 | -0.0010 |  |  |  |
| 280 | 0.030 | 0.00020 | 860 | 0.98 | -0.0010 |  |  |  |
| 290 | 0.032 | 0.00020 | 870 | 0.97 | -0.0013 |  |  |  |
| 300 | 0.034 | 0.00013 | 880 | 0.95 | 0.0000 |  |  |  |
| 310 | 0.035 | 0.00010 | 890 | 0.95 | 0.0000 |  |  |  |
| 320 | 0.036 | 0.00000 | 900 | 0.94 | 0.0000 |  |  |  |
| 330 | 0.036 | 0.00000 | 910 | 0.95 | 0.0000 |  |  |  |
| 340 | 0.037 | 0.00000 | 920 | 0.95 | 0.0000 |  |  |  |
| 350 | 0.036 | -0.00010 | 930 | 0.96 | 0.0010 |  |  |  |
| 360 | 0.035 | -0.00010 | 940 | 0.97 | 0.0010 |  |  |  |
| 370 | 0.034 | -0.00013 | 950 | 0.98 | 0.0000 |  |  |  |
| 380 | 0.032 | -0.00020 | 960 | 0.98 | 0.0000 |  |  |  |
| 390 | 0.030 | -0.00020 | 970 | 0.99 | 0.0013 |  |  |  |

TABLE 5. Approximate differences $\left.\delta=\left(t_{90}-t_{27}\right)\right)^{\circ} \mathrm{C}$ and derivatives of these differences $d \delta / \mathrm{d}(\mathrm{T} / \mathrm{K})$ as a function of $t_{27}{ }^{\circ} \mathrm{C}$. For $t_{27} 7^{\circ} \mathrm{C}<630$, the values given in Table 4 for $t_{48} /^{\circ} \mathrm{C}$ should be used.

| $t_{27}{ }^{\circ} \mathrm{C}$ | $\delta$ | $\mathrm{d} 8 / \mathrm{d}(T / \mathrm{K})$ | $t_{27} 7^{\circ} \mathrm{C}$ | $\delta$ | $\mathrm{d} \delta / \mathrm{d}(T / \mathrm{K})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 630 | 0.08 | 0.012 | 1000 | 1.25 | 0.0000 |
| 640 | 0.19 | 0.011 | 1010 | 1.25 | 0.0000 |
| 650 | 0.30 | 0.011 | 1020 | 1.24 | -0.0013 |
| 660 | 0.42 | 0.011 | 1030 | 1.22 | -0.0013 |
| 670 | 0.52 | 0.010 | 1040 | 1.21 | -0.0010 |
| 680 | 0.63 | 0.010 | 1050 | 1.20 | -0.0013 |
| 690 | 0.73 | 0.010 | 1060 | 1.18 | -0.0024 |
| 700 | 0.83 | 0.010 | 1100 | 1.04 | -0.0021 |
| 710 | 0.93 | 0.0095 | 1200 | 0.90 | -0.0022 |
| 720 | 1.02 | 0.0079 | 1300 | 0.35 | -0.0049 |
| 730 | 1.09 | 0.0070 | 1400 | -0.09 | -0.0044 |
| 740 | 1.16 | 0.0070 | 1500 | -0.54 | -0.0050 |
| 750 | 1.23 | 0.0065 | 1600 | -1.09 | -0.0055 |
| 760 | 1.29 | 0.0040 | 1700 | -1.64 | -0.0064 |
| 770 | 1.32 | 0.0038 | 1800 | -2.40 | -0.0071 |
| 780 | 1.37 | 0.0038 | 1900 | -3.06 | -0.0075 |
| 790 | 1.40 | 0.0024 | 2000 | -3.92 | -0.0081 |
| 800 | 1.42 | 0.0020 | 2100 | -4.69 | -0.0081 |
| 810 | 1.44 | 0.0000 | 2200 | -5.55 | -0.0092 |
| 820 | 1.44 | 0.0000 | 2300 | -6.53 | -0.010 |
| 830 | 1.43 | 0.0000 | 2400 | -7.60 | -0.010 |
| 840 | 1.43 | 0.0000 | 2500 | -8.57 | -0.011 |
| 850 | 1.42 | -0.0010 | 2600 | -9.75 | -0.012 |
| 860 | 1.41 | -0.0013 | 2700 | -11.0 | -0.012 |
| 870 | 1.39 | -0.0024 | 2800 | -12.2 | -0.013 |
| 880 | 1.36 | 0.0000 | 2900 | -13.6 | -0.014 |
| 890 | 1.36 | 0.0000 | 3000 | -15.1 | -0.015 |
| 900 | 1.34 | -0.0013 | 3100 | -16.6 | -0.016 |
| 910 | 1.33 | -0.0010 | 3200 | -18.3 | -0.016 |
| 920 | 1.32 | 0.0000 | 3300 | -19.9 | -0.017 |
| 930 | 1.32 | 0.0000 | 3400 | -21.7 | -0.019 |
| 940 | 1.31 | -0.0010 | 3500 | -23.7 | -0.020 |
| 950 | 1.30 | -0.0013 | 3600 | -25.7 | -0.021 |
| 960 | 1.28 | -0.0013 | 3700 | -27.9 | -0.022 |
| 970 | 1.27 | 0.0000 | 3800 | -30.1 | -0.023 |
| 980 | 1.27 | 0.0000 | 3900 | -32.4 | -0.025 |
| 990 | 1.26 | -0.0010 | 4000 | -35.1 | -0.028 |

TABLE 6. The parameters $a_{i}, b_{i}, c_{i}$ in eqs. (3) to (8).

| $\boldsymbol{i}$ | $a_{i}$ | $b_{i}$ | $c_{i}$ |
| :---: | ---: | :--- | :--- |
|  | -0.005903 | 0 | -0.00317 |
| 0 | 0.008174 | -0.148759 | -0.97737 |
| 1 | -0.061924 | -0.267408 | 1.25590 |
| 2 | -0.193388 | 1.080760 | 2.03295 |
| 3 | 1.490793 | 1.269056 | -5.91887 |
| 4 | -9.83584768 | -4.089591 | -3.23561 |
| 5 | -1.871251 | 7.23364 |  |
| 6 | 25.271912 | 7.438081 | 5.04151 |
| 7 | -19.183815 | -3.536296 | 0 |
| 8 | 0 | 0 |  |
| 9 | -18.437089 | 0 | 0 |
| 10 | 07.000895 | 0 | 0 |
| 11 | -8.716324 | 0 | 0 |
| 12 |  |  |  |

TABLE 7. Approximate effects on thermodynamic properties (heat capacity, enthalpy, and entropy) of several substances due to the change in the temperature scale IPTS-68 to the ITS-90. $C_{\mathrm{p}}(\mathrm{x}), H(\mathrm{x})$, and $S(\mathrm{x})$ are, respectively, the heat capacity, enthalpy, and entropy based either on the IPTS-68 or on the ITS-90 scale.

| $10^{2}\left\{C_{p}(90)-C_{p}(68)\right\} / C_{p}(90)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| $T / K$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{A1}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| 16 | 0.18 | 0.19 | 0.23 | 0.23 | 0.23 | 0.23 |
| 40 | 0.02 | 0.02 | 0.05 | 0.05 | 0.05 | 0.05 |
| 100 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.01 |
| 200 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 |
| 300 | 0.03 | 0.02 | 0.03 | 0.03 | 0.03 | 0.02 |
| 400 | - | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| 500 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 650 | - | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 |
| 800 | - | - | 0.04 | 0.04 | 0.04 | 0.04 |
| 850 | - | - | 0.04 | 0.04 | 0.04 | 0.04 |
| 900 | - | - | 0.00 | 0.00 | 0.00 | 0.00 |
| 920 | - | - | -0.44 | -0.44 | -0.44 | -0.44 |
| 950 | - | - | -0.44 | -0.45 | -0.45 | -0.47 |
| 980 | - | - | -0.40 | -0.40 | -0.40 | -0.40 |
| 1000 | - | - | -0.25 | -0.24 | -0.25 | -0.25 |
| 1020 | - | - | -0.21 | -0.20 | -0.21 | -0.21 |
| 1050 | - | - | -0.01 | 0.00 | -0.01 | -0.01 |
| 1080 | - | - | 0.15 | 0.15 | 0.14 | 0.14 |
| 1100 | - | - | 0.34 | 0.34 | 0.34 | 0.34 |
| 1150 | - | - | 0.34 | 0.34 | 0.34 | 0.34 |
| 1200 | - | - | 0.20 | 0.20 | 0.20 | 0.20 |
| 1300 | - | - | 0.10 | 0.10 | 0.10 | 0.11 |
| 1600 | - | - | 0.05 | 0.05 | 0.05 | 0.05 |
| 2000 | - | - | 0.06 | 0.07 | 0.07 | 0.08 |
| 2150 | - | - | 0.06 | 0.07 | 0.07 | 0.08 |
| 2400 | - | - | - | - | - | 0.10 |
| 2600 | - | - | - | - | - | 0.12 |
| 2800 | - | - | - | - | - | 0.15 |

$$
10^{2}\{H(90)-H(68)\} / H(90)
$$

$T / \mathrm{K} \quad \mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s}) \quad \mathrm{BaSnF}(\mathrm{s}) \quad \alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s}) \quad \mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s}) \quad \mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s}) \quad \mathrm{Mo}(\mathrm{s})$

| 16 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 | 0.08 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 40 | 0.04 | 0.04 | 0.07 | 0.06 | 0.06 | 0.06 |
| 100 | -0.02 | -0.02 | -0.04 | -0.04 | -0.03 | -0.03 |
| 200 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.01 |
| 300 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.00 |
| 400 | - | 0.01 | 0.02 | 0.02 | 0.02 | 0.01 |
| 500 | - | 0.01 | 0.02 | 0.02 | 0.02 | 0.01 |
| 650 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 800 | - | - | 0.02 | 0.02 | 0.02 | 0.01 |
| 850 | - | - | 0.02 | 0.02 | 0.02 | 0.02 |
| 900 | - | - | 0.02 | 0.02 | 0.02 | 0.02 |
| 920 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 950 | - | - | -0.02 | -0.01 | -0.01 | -0.01 |
| 980 | - | - | -0.03 | -0.03 | -0.03 | -0.03 |

TABLE 7 (continued)

| $10^{2}(H(90)-H(68)\} / H(90)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T / K$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{A1}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| 1000 | - | - | -0.04 | -0.04 | -0.04 | -0.04 |
| 1020 | - | - | -0.05 | -0.05 | -0.05 | -0.04 |
| 1050 | - | - | -0.05 | -0.05 | -0.05 | -0.04 |
| 1080 | - | - | -0.04 | -0.04 | -0.04 | -0.04 |
| 1100 | - | - | -0.03 | -0.03 | -0.03 | -0.03 |
| 1150 | - | - | -0.01 | -0.01 | -0.01 | -0.01 |
| 1200 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 1300 | - | - | 0.02 | 0.02 | 0.02 | 0.02 |
| 1600 | - | - | 0.03 | 0.03 | 0.03 | 0.03 |
| 2000 | - | - | 0.04 | 0.04 | 0.04 | 0.04 |
| 2150 | - | - | 0.04 | 0.04 | 0.04 | 0.04 |
| 2400 | - | - | - | - | - | 0.05 |
| 2600 | - | - | - | - | - | 0.06 |
| 2800 | - | - | - | - | - | 0.06 |


| $10^{2}(S(90)-S(68)\} / S(90)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T / K$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| 16 | 0.11 | 0.11 | 0.09 | 0.11 | 0.12 | 0.05 |
| 40 | 0.03 | 0.03 | 0.05 | 0.05 | 0.06 | 0.05 |
| 100 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | 0.00 |
| 200 | -0.01 | 0.00 | -0.02 | -0.02 | -0.02 | -0.01 |
| 300 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 400 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 500 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 650 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 800 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 850 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 900 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 920 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 950 | - | - | 0.00 | 0.01 | 0.00 | 0.00 |
| 980 | - | - | -0.01 | -0.01 | -0.01 | -0.01 |
| 1000 | - | - | -0.02 | -0.01 | -0.01 | -0.01 |
| 1020 | - | - | -0.02 | -0.01 | -0.01 | -0.01 |
| 1050 | - | - | -0.02 | -0.01 | -0.01 | -0.01 |
| 1080 | - | - | -0.02 | -0.01 | -0.01 | -0.01 |
| 1100 | - | - | -0.01 | -0.01 | -0.01 | -0.01 |
| 1150 | - | - | 0.00 | 0.00 | 0.00 | 0.00 |
| 1200 | - | - | 0.01 | 0.01 | 0.01 | 0.01 |
| 1300 | - | - | 0.01 | 0.02 | 0.02 | 0.01 |
| 1600 | - | - | 0.02 | 0.02 | 0.02 | 0.01 |
| 2000 | - | - | 0.02 | 0.02 | 0.02 | 0.02 |
| 2150 | - | - | 0.02 | 0.03 | 0.03 | 0.02 |
| 2400 | - | - | - | - | - | 0.02 |
| 2600 | - | - | - | - | - | 0.03 |
| 2800 | - | - | - | - | - | 0.03 |

TABLE 8. Approximate effects on thermodynamic properties (heat capacity, enthalpy, and entropy) of several substances due to the change in the temperature scale IPTS-48 to the ITS-90. $C_{p}(\mathrm{x}), H(\mathrm{x})$, and $S(\mathrm{x})$ are, respectively, the heat capacity, enthalpy, and entropy based either on the IPTS-48 or on the ITS-90 scale.

| $10^{2}\left\{C_{p}(90)-C_{p}(48)\right\} / C_{p}(90)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| $T / \mathrm{K}$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| 100 | 0.02 | 0.03 | -0.01 | -0.01 | 0.00 | 0.02 |
| 150 | -0.10 | -0.10 | -0.11 | -0.11 | -0.11 | -0.10 |
| 200 | 0.00 | -0.01 | -0.03 | -0.03 | -0.03 | -0.01 |
| 250 | 0.08 | 0.08 | 0.07 | 0.07 | 0.07 | 0.08 |
| 300 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| 350 | - | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 |
| 400 | - | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| 500 | - | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| 650 | - | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| 800 | - | - | -0.01 | -0.01 | -0.01 | -0.01 |
| 850 | - | - | -0.06 | -0.08 | -0.08 | -0.06 |
| 900 | - | - | -0.16 | -0.16 | -0.16 | -0.16 |
| 920 | - | - | -0.70 | -0.70 | -0.70 | -0.70 |
| 950 | - | - | -0.82 | -0.82 | -0.82 | -0.82 |
| 980 | - | - | -0.71 | -0.71 | -0.71 | -0.71 |
| 1000 | - | - | -0.48 | -0.48 | -0.49 | -0.49 |
| 1020 | - | - | -0.51 | -0.51 | -0.51 | -0.51 |
| 1050 | - | - | -0.33 | -0.33 | -0.33 | -0.34 |
| 1080 | - | - | -0.10 | -0.10 | -0.10 | -0.10 |
| 1100 | - | - | 0.08 | 0.09 | 0.08 | 0.08 |
| 1150 | - | - | 0.20 | 0.21 | 0.20 | 0.19 |
| 1200 | - | - | -0.15 | -0.14 | -0.15 | -0.15 |
| 1600 | - | - | -0.16 | -0.16 | -0.17 | -0.19 |
| 2000 | - | - | -0.15 | -0.16 | -0.16 | -0.21 |
| 2150 | - | - | -0.15 | -0.16 | -0.16 | -0.22 |
| 2400 | - | - | - | - | - | -0.24 |
| 2600 | - | - | - | - | - | -0.27 |
| 2800 | - | - | - | - | - | -0.49 |

$$
10^{2}\{H(90)-H(48)\} / H(90)
$$

| $T / \mathrm{K}$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| 100 | -0.04 | -0.04 | -0.07 | -0.07 | -0.07 | -0.05 |
| 150 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 | -0.02 |
| 200 | -0.04 | -0.04 | -0.07 | -0.07 | -0.06 | -0.04 |
| 250 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.01 |
| 300 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 350 | - | 0.01 | 0.02 | 0.02 | 0.02 | 0.01 |
| 400 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 500 | - | 0.00 | -0.01 | -0.01 | -0.01 | 0.00 |
| 650 | - | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| 800 | - | - | 0.00 | 0.00 | 0.00 | 0.00 |
| 850 | - | - | 0.00 | 0.00 | 0.00 | 0.00 |
| 900 | - | - | -0.01 | -0.01 | -0.01 | -0.01 |
| 920 | - | - | -0.03 | -0.03 | -0.03 | -0.03 |
| 950 | - | - | -0.06 | -0.07 | -0.07 | -0.06 |
| 980 | - | - | -0.09 | -0.09 | -0.09 | -0.08 |

TABLE 8 (continued)

| $10^{2}(H(90)-H(48)\} / H(90)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T / K$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| 1000 | - | - | -0.11 | -0.11 | -0.11 | -0.09 |
| 1020 | - | - | -0.12 | -0.12 | -0.12 | -0.10 |
| 1050 | - | - | -0.13 | -0.13 | -0.13 | -0.11 |
| 1080 | - | - | -0.13 | -0.13 | -0.13 | -0.12 |
| 1100 | - | - | -0.13 | -0.13 | -0.13 | -0.11 |
| 1150 | - | - | -0.12 | -0.12 | -0.11 | -0.10 |
| 1200 | - | - | -0.11 | -0.11 | -0.11 | -0.10 |
| 1600 | - | - | -0.12 | -0.12 | -0.12 | -0.12 |
| 2000 | - | - | -0.13 | -0.13 | -0.13 | -0.14 |
| 2150 | - | - | -0.13 | -0.14 | -0.14 | -0.15 |
| 2400 | - | - | - | - | - | -0.16 |
| 2600 | - | - | - | - | - | -0.18 |
| 2800 | - | - | - | - | - | -0.20 |


| $10^{2}\{S(90)-S(48)\} / S(90)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T / \mathrm{K}$ | $\mathrm{ND}_{4} \mathrm{ReO}_{4}(\mathrm{~s})$ | $\mathrm{BaSnF}_{4}(\mathrm{~s})$ | $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{BeO} \cdot 3 \mathrm{Al}_{2} \mathrm{O}_{3}(\mathrm{~s})$ | $\mathrm{Mo}(\mathrm{s})$ |
| 100 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | 0.00 |
| 150 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | 0.00 |
| 200 | -0.02 | -0.02 | -0.05 | -0.05 | -0.05 | -0.02 |
| 250 | -0.01 | 0.00 | -0.02 | -0.02 | -0.02 | -0.01 |
| 300 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 |
| 350 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 400 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 500 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 650 | - | 0.00 | -0.01 | -0.01 | -0.01 | 0.00 |
| 800 | - | - | 0.00 | 0.00 | 0.00 | 0.00 |
| 850 | - | - | 0.00 | 0.00 | 0.00 | 0.00 |
| 900 | - | - | -0.01 | -0.01 | -0.01 | 0.00 |
| 920 | - | - | -0.01 | -0.02 | -0.02 | -0.01 |
| 950 | - | - | -0.03 | -0.03 | -0.03 | -0.02 |
| 980 | - | - | -0.04 | -0.04 | -0.04 | -0.03 |
| 1000 | - | - | -0.05 | -0.05 | -0.05 | -0.03 |
| 1020 | - | - | -0.06 | -0.06 | -0.05 | -0.04 |
| 1050 | - | - | -0.06 | -0.06 | -0.06 | -0.04 |
| 1080 | - | - | -0.07 | -0.06 | -0.06 | -0.04 |
| 1100 | - | - | -0.06 | -0.06 | -0.06 | -0.04 |
| 1150 | - | - | -0.06 | -0.06 | -0.06 | -0.04 |
| 1200 | - | - | -0.06 | -0.06 | -0.05 | -0.04 |
| 1600 | - | - | -0.08 | -0.07 | -0.07 | -0.06 |
| 2000 | - | - | -0.09 | -0.08 | -0.08 | -0.07 |
| 2150 | - | - | -0.09 | -0.09 | -0.09 | -0.08 |
| 2400 | - | - | - | - | - | -0.09 |
| 2600 | - | - | - | - | - | -0.09 |
| 2800 | - | - | - | - | - | -0.11 |
|  |  |  |  |  |  |  |

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## REFERENCES

1. Procès-verbaux du Comité International des Poids et Mesures, Bureau International des Poids et Mesures, Pavillon de Breteuil, Sèvres, France (1990).
2. H. Preston-Thomas, Metrologia 27, 3-10 (1990); Metrologia 27, 107 (1990).
3. M.L. McGlashen, J. Chem. Thermodyn. 22, 653-663 (1990).
4. H. Preston-Thomas, P. Bloembergen, and T.J. Quinn, Supplementary Information for the International Temperature Scale of 1990, Bureau International des Poids et Mesures, Pavillon de Breteuil, Sèvres, France (1990).
5. R.E. Bedford, G. Bonnier, H. Maas and F. Pavese, Techniques for Approximating the International Temperature Scale of 1990, Bureau International des Poids et Mesures, Pavillon de Breteuil, Sèvres, France (1990).
6. B.W. Mangum and G.T. Furukawa, Guidelines for Realizing the International Temperature Scale of 1990 (ITS-90), NIST Technical Note 1265, National Institute of Standards and Technology, Gaithersburg, Maryland (1990).
7. H. Preston-Thomas, P. Bloembergen, and T. Quinn, Supplementary Information for the IPTS-68 and the EPT-76, Bureau International des Poids et Mesures, Pavillon de Breteuil, Sèvres, France (1983).
8. NAG Fortran Library Manual - Mark 13, Numerical Algorithms Group, Ltd., Oxford (1988).
9. R. Boisvert, S.E. Howe, D.K. Kahaner, and J.L. Springmann, Guide to Available Mathematical Software, NBSIR 90-4237, National Institute of Standards and Technology, Gaithersburg, Maryland (1990).
10. J.L. Riddle, G.T. Furukawa, and H.H. Plumb, Platinum Resistance Thermometry, Natl. Bur. Stand. Monograph 126, U.S. Government Printing Office, Washington, D.C. (1972).
11. R.L. Rusby, J. Chem. Thermodyn. 23, 1153-1162 (1991).
12. T.B. Douglas, J. Res. Natl. Bur. Stand., Sect. A 73, 451-470 (1969).
13. R.J.C. Brown, J.E. Callanan, R.D. Weir, and E.F. Westrum, Jr., J. Chem. Thermodyn. 18, 787-792 (1986).
14. J.E. Callanan, R.D. Weir, and E.F. Westrum, Jr., Internat. J. Thermophys. 9, 1091-1100 (1988).
15. J.E. Callanan and R.D. Weir, J. Chem. Thermodyn. 23, 411-420 (1991).
16. D.A. Ditmars, S. Ishihara, S.S. Chang, G. Bernstein, and E.D. West, J. Res. Natl. Bur. Stand. 87, 159-163 (1982).
17. S.S. Chang, Proceedings of the Seventh Symposium on Thermophysical Properties, ASME, held at the National Bureau of Standards, Gaithersburg, Maryland, May 10-12, 1977, pp. 83-90.
18. D.A. Ditmars and T.B. Douglas, J. Res. Natl. Bur. Stand., Sect. A 75, 401-420 (1971).
19. G.T. Furukawa and W.G. Saba, J. Res. Natl. Bur. Stand., Sect. A 69, 13-18 (1965).
20. D.A. Ditmars and T.B. Douglas, J. Res. Natl. Bur. Stand., Sect. A 71, 89-95 (1967).
21. G.T. Furukawa and W.G. Saba, J. Res. Natl. Bur. Stand., Sect. A 71, 3-8 (1967).
22. D.A. Ditmars and T.B. Douglas, J. Res. Natl. Bur. Stand., Sect. A 71, 97-103 (1967).
23. K. Klusius and P. Franzosini, Z. Naturforsch., A: Phys. Phys. Chem. Kosmophys. 14, 991-105 (1959).
24. D.A. Ditmars, A. Cezairliyan, S. Ishihara, and T.B. Douglas, Enthalpy and Heat Capacity Standard Reference Material: Molybdenum SRM 781, from 273 to 2800 K, Natl. Bur. Stand. Spec. Pub. 260-55, U.S. Government Printing Office, Washington, D.C. (1977).
25. G.T. Furukawa and M.L. Reilly, J. Res. Natl. Bur. Stand., Sect. A 69, 5-12 (1965).
26. G.T. Furukawa and M.L. Reilly, in Temperature: Its Measurement and Control in Science and Industry, Vol. 4, (H.H. Plumb, editor-in-chief), pp. 27-36 Instrument Society of America, Pittsburgh, Pennsylvania (1972).
27. D.C. Ginnings, in Precision Measurement and Calibration: Selected NBS Papers on Heat, Natl. Bur. Stand. Spec. Pub. 300 - Vol. 6, (D.C. Ginnings, editor), pp. 1-14, U.S. Government Printing Office, Washington, D.C. (1970).
28. N.S. Osborne, H.F. Stimson, and D.C. Ginnings, J. Res. Natl. Bur. Stand. (U.S.) 23, 197-270 (1939).
29. G.W. Burns, G.F. Strouse, B.W. Mangum, M.C. Croarkin, W.F. Guthrie, P. Marcarino, M. Battuello, H.K. Lee, J.C. Kim, K.S. Gam, C. Rhee, M. Chattle, M. Arai, H. Sakurai, A.I. Pokhodun, N.P. Moiseeva, S.A. Perevalova, M.J. deGroot, J. Zhang, K. Fan, and S. Wu, New reference function for platinum-10\% rhodium versus platinum (type S) thermocouples based on the ITS-90. Part I: Experimental procedures, in Temperature: Its Measurement and Control in Science and Industry, Vol. 6 (J.F. Schooley, editor), American Institute of Physics, New York, in press.
30. G.W. Burns, G.F. Strouse, B.W. Mangum, M.C. Croarkin, W.F. Guthrie, P. Marcarino, M. Battuello, H.K. Lee, J.C. Kim, K.S. Gam, C. Rhee, M. Chattle, M. Arai, H. Sakurai, A.I. Pokhodun, N.P. Moiseeva, S.A. Perevalova, M.J. deGroot, J. Zhang, K. Fan, and S. Wu, New reference function for platinum-10\% rhodium versus platinum (type S) thermocouples based on the ITS-90. Part II: Results and discussion, in Temperature: Its Measurement and Control in Science and Industry, Vol. 6 (J.F. Schooley, editor), American Institute of Physics, New York, in press.
31. W.F. Guthrie, M.C. Croarkin, G.W. Burns, G.F. Strouse, P. Marcarino, M. Battuello, H.K. Lee, J.C. Kim, K.S. Gam, C. Rhee, M. Chattle, M. Arai, H. Sakurai, A.I. Pokhodun, N.P. Moiseeva, S.A. Perevalova, M.J. deGroot, J. Zhang, K. Fan, and S. Wu, Statistical analysis of type S thermocouple measurements on the International Temperature Scale of 1990, in Temperature: Its Measurement and Control in Science and Industry, Vol. 6 (J.F. Schooley, editor), American Institute of Physics, New York, in press.

[^0]:    ${ }^{\dagger}$ A second-order correction to the differences is needed due to the shift in the independent variable from $T_{90} / \mathrm{K}$ (or $t_{90}{ }^{\circ} \mathrm{C}$ ) to $T_{68} / \mathrm{K}$ (or $t_{68}{ }^{\circ} \mathrm{C}$ ). However, this second-order correction was found in all cases to be less than the least significant figure given for the differences between the two temperature scales. A point related to the construction of all of the tables and equations given in this paper pertains to the notation used for temperature. When it is necessary to specify the temperature scale, it is current practice to attach a subscript to the thermodynamic temperature ( $T$ ) and to the Celsius temperature ( $t$ ). Thus, the temperature on the ITS 90 is expressed as $T_{90}$ or $t_{90}$. The kelvin is kept fixed at exactly $1 / 273.16$ of the thermodynamic temperature of the triple point of water. The Celsius temperature is defined by: $t /{ }^{\circ} \mathrm{C}=$ T/K - 273.15.

